

Prediction Equations to Determine the Induced Force on Reinforcements due to
Laterally Loaded Piles Behind MSE Walls

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ABSTRACT

Researchers performed 35 full scale lateral load tests on piles driven within the reinforcement zone of an MSE wall. Data defining the induced tensile force on the reinforcements during lateral pile loading was used to develop multiple-linear regression equations to predict the induced tensile force. Equations were developed by previous researchers that did not consider, diameter of the pile, fixed head condition, relative compaction, and cyclic loading. The purpose of this research was to include all available data and develop prediction equations that considered the variables previously mentioned.

The diameter of the pile is a statistically significant variable for the prediction of induced tensile force, the induced tensile force is lower for piles with larger diameters. Fixed head conditions were found to have no significant effect on the prediction of induced tensile force. In addition, cyclic loading did not have a large impact on the prediction of induced tensile force but relative compaction did have an important statistical significance. Prediction equations for induced tensile force in welded wire reinforcements were developed for relative compaction less than 95 percent and relative compaction greater or equal to 95 percent. A general prediction equation (Eq. 3-4) was developed for ribbed-strip reinforcements that included the effect of pile diameter and larger pile head loads. With 1058 data points, this equation had an R^2 value of 0.72. A general prediction equation (Eq. 3-9) was also developed for welded-wire reinforcements that included data from cyclic and static loading, fixed and free head conditions, and relative compaction for 12-inch wide piles with a higher range of pile head loads. This equation based on 2070 data points had an R^2 value of 0.72. The prediction equations developed based on all the available data are superior to equations developed based on the original set of field tests.

Keywords: MSE wall, laterally loaded piles, relative compaction, reinforcement force

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1 INTRODUCTION

Mechanically stabilized earth (MSE) walls have been used widely in the construction industry and their use as a bridge abutment is very common. Additionally, piles within the reinforcement zones of MSEW for abutments are often a solution for a foundation system. Piles must be designed to support axial load as well as lateral loads from thermal expansion and contraction, earthquakes, and traffic loads. Dr Kyle Rollins' research team developed two multiple linear regression equations to predict the maximum induced tensile force in reinforcing elements in MSE walls when piles are loaded laterally (Luna, 2016); however, these equations were developed for a limited range of pile diameters, pile head loads and pile head fixity conditions. Additional full-scale testing (Wilson 2020 and Flores, 2020) suggest that these equations should be updated to appropriately account for a wider range of loading conditions.

Pile caps and bridge abutments, that can be approximated by a fixed head boundary condition are very common in bridge design. Most of the time, piles have caps to help transfer loads from the structure to the foundations. In contrast, the lateral pile load tests used by Luna (2016) to develop the equations for induced force in the reinforcement were all free-head piles. Therefore, the effect of a fixed head boundary condition on the induced reinforcement force has not been considered. Additionally, for the development of the prediction equations, most of the load tests were performed with 12.75-inch diameter piles, within soil having a relative compaction 90% of the stand Proctor density, and all the tests were performed for static loading case. Therefore, using piles of greater diameter, with different relative compaction, and cyclic loading could have a considerable impact in the prediction of the induced forces in the reinforcing elements.

Understanding these conditions will help optimize the design of bridge abutments with MSE wall.

In the following study, data from additional large scale lateral load tests on piles near MSE walls will be used to ameliorate prediction equations for the maximum induced tensile force in reinforcing elements. These tests involve fixed head loading, cyclic loading, 24-inch diameter piles and backfill compacted to 95% of the standard Proctor density.

1.1 Objectives

The objectives of this study are the following:

1. Develop correlation equations to predict induced tensile force in reinforcing elements for MSEW from piles installed within the reinforcement zone that are laterally loaded.
2. Study the effect of relative compaction, pile head fixity, cyclic loading, and larger diameter piles on the induced tensile force in reinforcing elements in MSE wall when piles behind the wall are laterally loaded.

1.2 Scope

Researchers have already developed two prediction equations for reinforcing elements in MSE walls due to laterally loaded piles within the reinforcement zone (Luna, 2016). Data for these equations were obtained from full-scale tests. These equations were developed for welded wire reinforcement and ribbed strip reinforcements where the reinforced soil was compacted to around 90% relative compaction based on a standard Proctor test. This study will add into the statistical data for induced tensile force in the reinforcing elements from full-scale lateral load tests performed on 24 inch diameter piles, cyclically loaded piles (Wilson, 2020), and piles with fixed head condition. In addition, the relative compaction of the soil will be taken into consideration. Multilinear regression analyses will be performed to develop equations to predict

the induced tensile force for each of the various loading conditions, Finally, attempts will be made to develop a single equation that can predict the induced tensile force for all the loading conditions for the two separate steel reinforcement types. The statistical significance of different variables will be studied to determine the effect of the mentioned conditions.

1.3 Outline

This study shows full scale tests data of laterally loaded piles behind MSE wall for piles and reinforcing soil elements. It has data from strained ribbed strip reinforcement and welded wire reinforcement in MSE wall that were strained during the load tests. Moreover, it shows a statistical analysis to select variables that are statistically significant and the development for a multiple linear regression for ribbed strip reinforcements and welded wire separately using data from phase 1, phase 2 of the tests to make the input is dimensionless, and phase 3 to include data from laterally loaded large diameter piles. Additionally, Phase 3 prediction equations will be tested on group piles data.

Furthermore, a prediction equation for welded wire reinforcement will be develop for phase 1 and phase 2 to make the input dimensionless. Data from phase 3 is added into the prediction equation which includes piles that were laterally loaded with a pile cap with the intent of including fixed head conditions. Cyclically loaded piles test data will be also compared with the developed prediction equation and an equation that includes cyclically loaded piles was developed.

2 BACKGROUND INFORMATION

2.1 Laterally loaded piles behind MSEW

Dr Kyle Rollins' research team performed full-scale tests that were divided into 3 phases. Phase 1 included full-scale load tests on piles that were laterally loaded behind MSE walls that had a height of 15 ft, and relative compaction of the soil between the piles and the face of the wall was of about 90%. It included cases with the following conditions: pipe piles and H piles in the reinforced area with ribbed strip reinforcement and square and pipe piles in the reinforced area with welded wire reinforcement. Figure 2-1 shows a plan view of the MSE wall with the different conditions described for phase 1. Phase 2 increased the height of the wall to 20 ft.

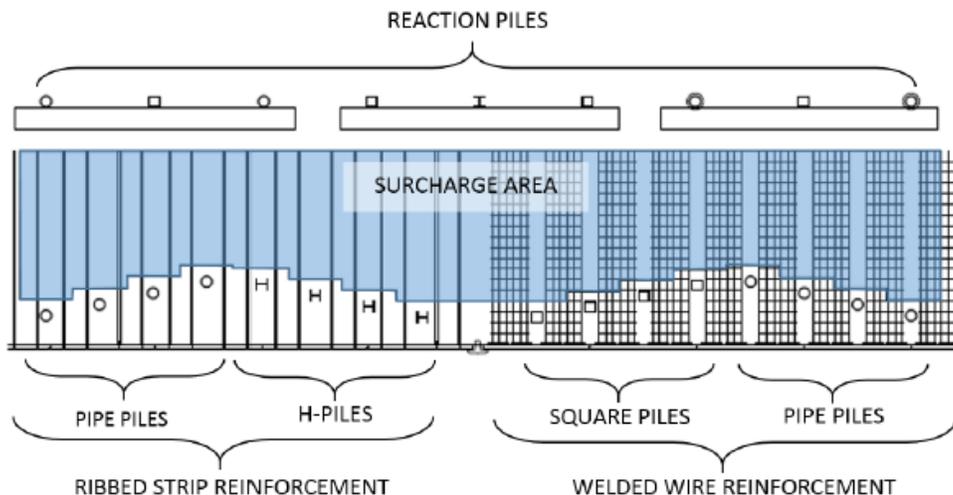


Figure 2-1. Plan view of the MSEW for Phase 1 test loads (Luna, 2016)

For phase 3 of the load tests the top 6.25 ft of the MSE wall was replaced with a more compacted soil that achieved a relative compaction between the piles and the face of the wall of about 95%. 24-inch diameter piles and piles that were loaded as a group were placed in the ribbed strip reinforced zone and capped piles with cyclically loaded piles were placed in the welded wire reinforced area. Figure 2-2 shows a plan view of the MSE wall for these tests.

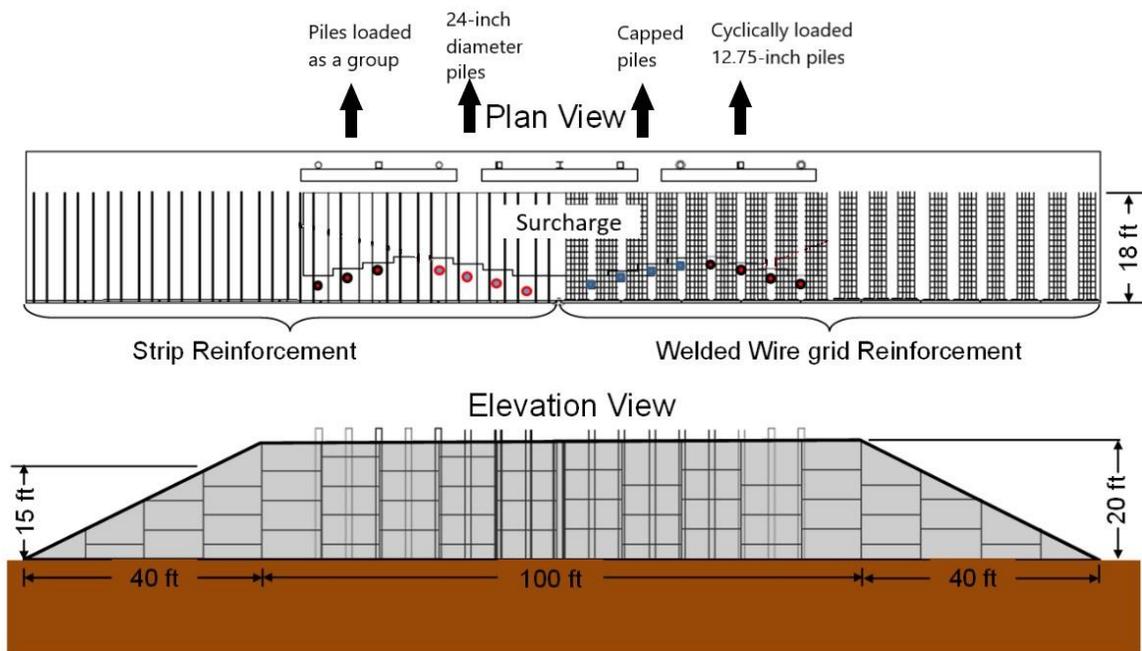


Figure 2-2. Plan view of the MSEW for phase 3

Strain gauges were placed on the reinforcing elements to measure the strain that they experienced when the piles were loaded laterally. Researchers (Han, 2014), observed an increase in the maximum tensile force experienced by the reinforcing element as the lateral load applied to the head of the pile increased. They also observed that as the spacing between the piles and the wall decreased the maximum induced force increased. This could be because as the spacing

between the wall and the pile decreases, the volume of soil resisting lateral pile deflection decreases and there is less soil to take stresses, thus, the reinforcement equilibrates these stresses.

A schematic drawing illustrating the likely behavior of the pile-soil-reinforcement interaction is presented in Figure 2-3. The force distribution in the reinforcement suggests that soil in front of the pile is being pushed forward as the pile is loaded while soil behind the pile is resisting movement of the reinforcement. In front of the pile, the soil is moving toward the wall relative to the reinforcement. This leads to an increase in tension in the reinforcement, moving from the wall to the pile, as load is transferred from the soil to the reinforcement by skin friction. Any positive tensile force in the reinforcement at the wall face is likely a result of the increased earth pressure on the wall. Behind the pile, the reinforcement is moving towards the wall relative to the soil. This leads to a decrease in tension in the reinforcement, moving from the pile to the end of the reinforcement, as load is transferred to the surrounding soil by skin friction.

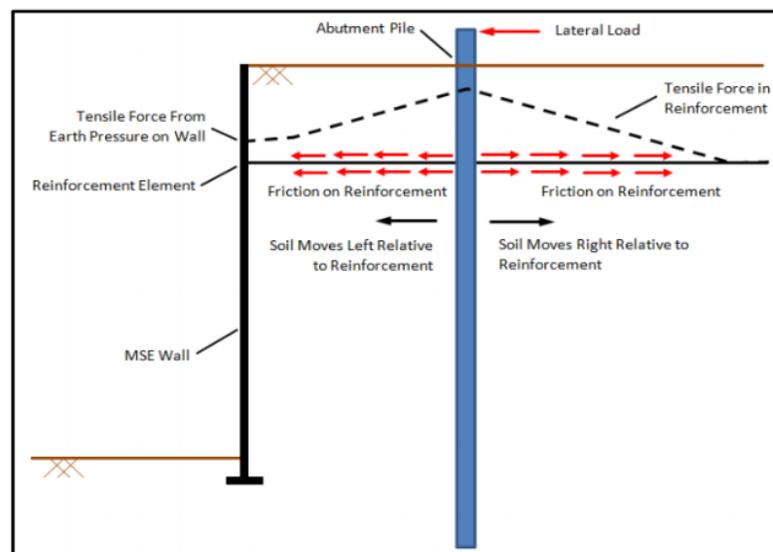


Figure 2-3. Interaction between soil and soil reinforcements when a pile is laterally loaded (Han, 2014)



Figure 2-4. Instrumentation of a ribbed strip reinforcement

The tensile force in the soil reinforcements were calculated with data recorded from strain gauges. Ribbed strip reinforcements were instrumented with strain gauges on both the top and bottom. The average strain reading was then used to calculate the induced force in the reinforcement at various distances from the back face of the MSE wall. Occasionally, strain gauges were damaged or failed to give accurate readings, when this happened induced tensile force was based only on one value. The measurements were given in micro-strain and the induced tensile force in the reinforcing soil elements was calculated with the following equations:

- For ribbed strip reinforcements:

$$F = EA \left(\frac{\mu\epsilon_t + \mu\epsilon_b}{n} \right) (10^{-6}) \quad (2-1)$$

- For welded wire reinforcements:

$$F = EA (\mu\epsilon_{AVG})(10^{-6})(B - 1) \quad (2-2)$$

Where:

F is the induced reinforcement load for a given reinforcement grid or a ribbed strip reinforcement in kips,

E is the modulus of elasticity, 2,900 ksi,

A is the cross-sectional area of a single welded wire (longitudinal), 0.11 in² or the cross-sectional area of a ribbed strip reinforcement, 0.32 in²

n is equal to 1 when one strain reading is omitted and equal to 2 when neither is omitted,

$\mu\epsilon_t$ is the micro strain of the top gauge,

$\mu\epsilon_b$ is the micro strain of the bottom gauge,

$\mu\epsilon_{AVG}$ is the average micro strain of the top and bottom strain gauges

B is the number of longitudinal wires in the reinforcement grid.

For example, for a ribbed strip reinforcement:

A strain gauge measurement of 234.11 μ was taken from the top of the reinforcement.

$$F = (2900)(0.32) \left(\frac{234.11}{1} \right) 10^{-6} = 2.14 \text{ kips}$$

For a welded wire grid reinforcement with an average strain measurement of 204.04 μ and five longitudinal bars,

$$F = (2900)(0.11)(4)(204.04)10^{-6} = 2.6 \text{ kips}$$

2.2 Maximum tensile force prediction equation on reinforcing elements of MSE wall

Due to the relatively complicated soil structure interaction between the pile, backfill, wall, and soil reinforcement, researchers were unable to develop a simple equation to predict forces induced in the soil reinforcements. Instead, regression equations were produced using the Statistical Analysis System (SAS) software program and the Data Analysis pack for Microsoft® Excel. An effort was made to reduce as many parameters as possible without significantly decreasing the R^2 value for each model (Luna, 2016). Separate equations were developed for the ribbed strip and welded-wire reinforcement types because of the difference in geometry of the two reinforcements.

The most recent regression analysis of ribbed strip soil reinforcements used 942 data observations from previously performed studies, resulting in an R^2 value of 0.71. In this equation, the maximum induced force in a ribbed strip reinforcement due to a laterally loaded pile is calculated with the following prediction equation:

$$\Delta F(kips) = 10^{\left(0.13 + 0.028P - 2.2 \times 10^{-4}P^2 - 0.01 \frac{T}{D} - 0.0021P \frac{T}{D} - 0.031 \frac{S}{D}\right)} - 1 \quad (2-3)$$

Where:

F is the maximum predicted tensile force (kip),

P is the pile head load (kip),

T is the transverse distance from reinforcement to pile center (in.),

D is the pile diameter (in.), and

S is spacing from pile center to back face of MSE wall (in.).

Figure 2-5 shows the predicted induced load on the reinforcement using Equation 2-3 compared with the measured loads, (Luna, 2016). The solid line represents perfect agreement where the measured load equals the predicted load, while the dashed lines represent the mean plus and minus one and two standard deviations.

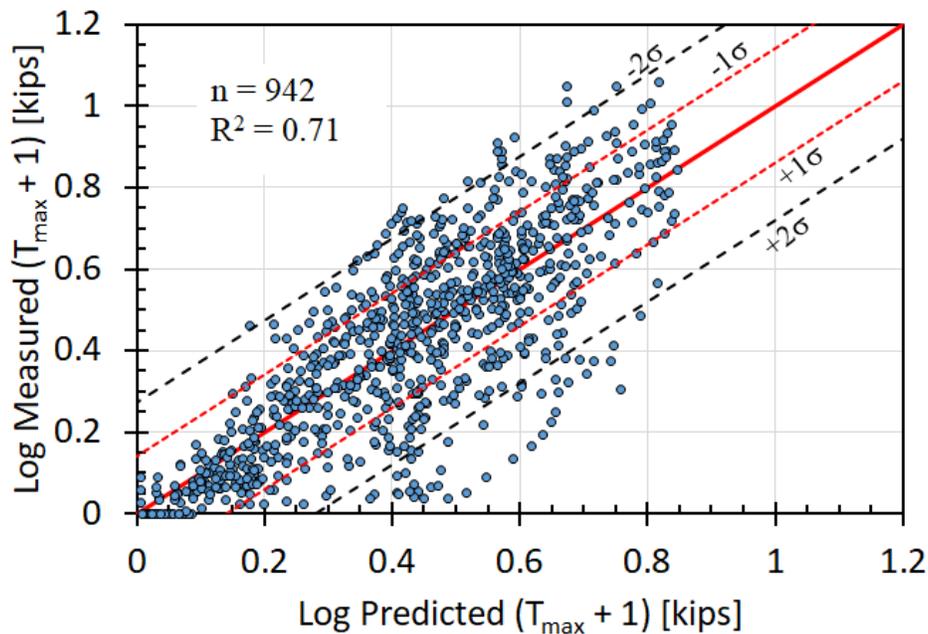


Figure 2-5. Predicted versus measured maximum ribbed strip reinforcement tensile force. (Luna, 2016)

The most recent regression analysis of welded wire soil reinforcements used 1,058 data observations from previously performed studies, resulting in an R^2 value of 0.72. In this equation, the maximum induced force in a ribbed strip reinforcement due to a laterally loaded pile is calculated with the following empirical equation:

$$\Delta F(kip) = 10^{(-0.04 + 0.027P - 2.7 \times 10^{-4}P^2 + 5.7 \times 10^{-4}\sigma_v - 2.6 \times 10^{-7}\sigma_v^2 - 0.08 \frac{T}{D})} - 1$$

(2-4)

where F = the maximum predicted tensile force (kip),

P = the pile head load (kip),

σ_v = the vertical stress (psf),

T = the transverse distance from reinforcement to pile center (in.), and

D = the pile diameter (in.).

In contrast to the equation for ribbed-strip reinforcements, the regression equation for welded wire soil reinforcement, Equation 2-4, considers the vertical effective stress acting at the level of each reinforcement. It is noted that unlike the regression equation for ribbed strip soil reinforcement, Equation 2-3, the equation for welded wire reinforcement does not consider the pile-to-wall spacing.

Figure 2-6 shows the predicted induced load on the reinforcement using Equation 2-4 compared with the observed loads, (Luna, 2016). The solid line represents the case where the measured load equals the predicted load, while the dashed lines represent the mean plus and minus one and two standard deviations.

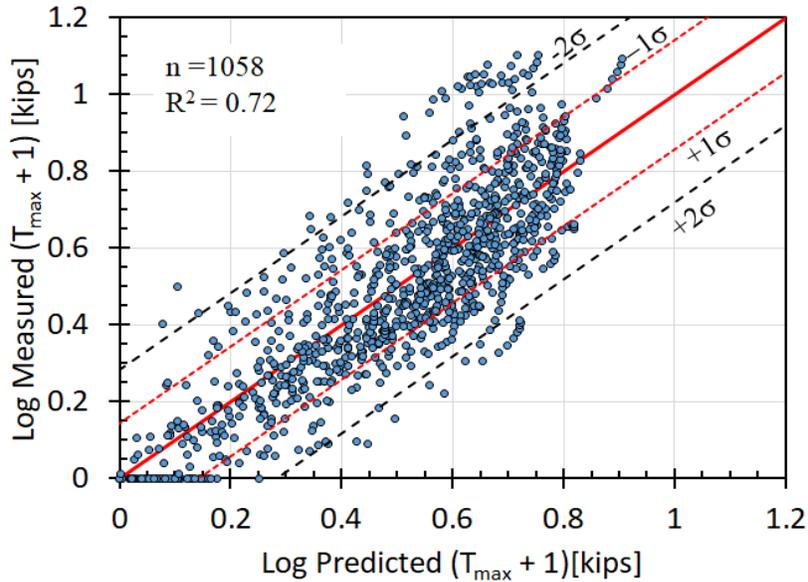


Figure 2-6. Predicted versus measured maximum welded wire reinforcement tensile force (Luna, 2016)

2.2.1 Using prediction equation on large diameter piles

During Phase 3 research, Dr. Rollins’ research team performed lateral load tests on 24-inch diameter piles behind an MSE wall. The soil compaction level was changed to more thoroughly investigate different conditions. The 24-inch diameter piles were driven at distances of 4, 6, 8, and 10 ft (or 2, 3, 4, and 5 pile diameters) behind the back face of the MSE to the center of the test piles. As mentioned before, the MSE wall height during these tests was 20 ft, reinforcements were 18 ft long, and soil in the top 6.25 ft from the surface was compacted to 95% relative to the standard Proctor between the face of the wall and the piles.

Strain gauges were attached to the ribbed stripped reinforcements for test as shown in Figure 2-7. The predicted maximum tensile force experienced by each instrumented

reinforcement during the four statically loaded pile tests was calculated using equation 2-4 for each load increment for each of the four test piles (Wilson, 2020).

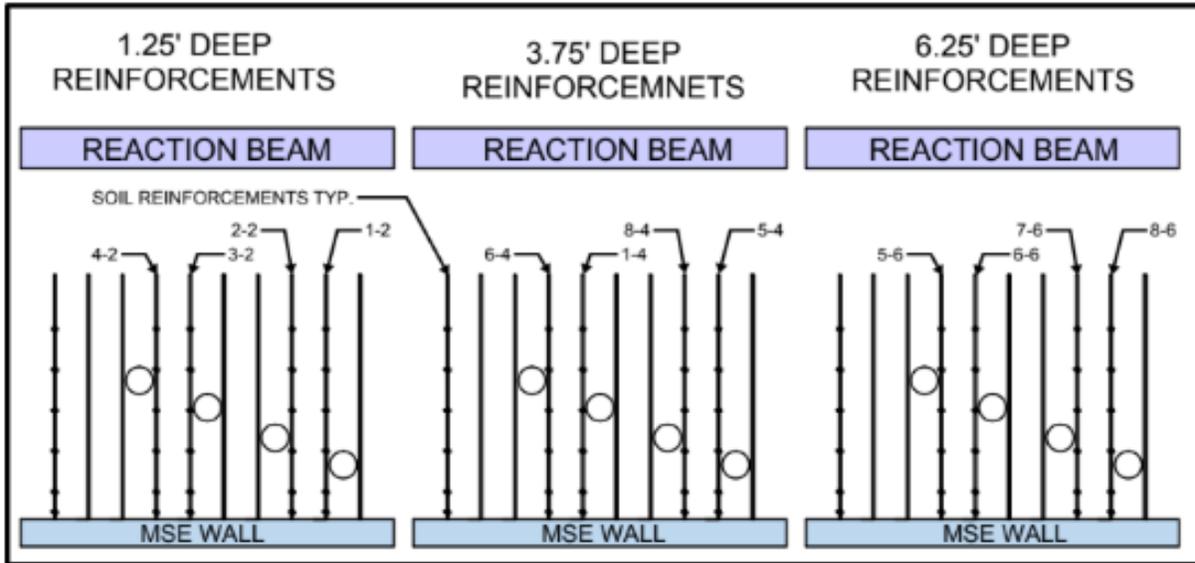


Figure 2-7. Map of the soil reinforcements near the 24-inch piles. (Wilson, 2020)

The predicted maximum tensile force was compared afterwards to the maximum observed tensile forces in the various reinforcements during lateral load testing of each pile. Figure 2-8 shows a comparison of the log of the measured maximum tensile force plus one relative to the log of computed maximum tensile force plus one. About 64% of the data points fell within one standard deviation boundary of the mean which is close to the 68.2% expected for a normal distribution; however the data points appear to be more evenly distributed than anticipated.

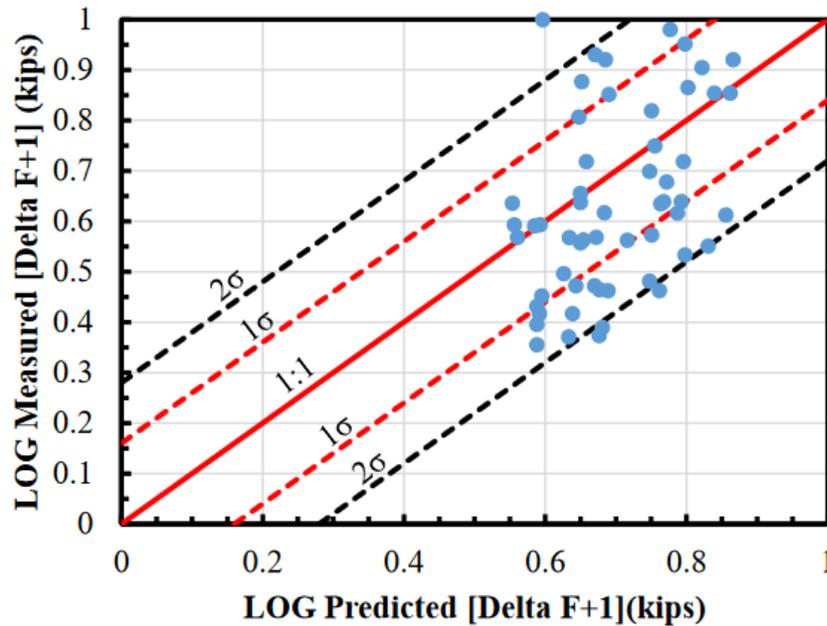


Figure 2-8. Statistical comparison of previously suggested equation for prediction of tensile force experienced by the soil reinforcements and the measured tensile force by ribbed strip reinforcements. (Wilson, 2020).

2.2.2 Using prediction equation on piles with fixed head conditions

Induced tensile force in the reinforcements was also evaluated in phase 3 of the tests laterally loaded test piles with a fixed head boundary condition. These tests involved four H-piles that were driven at distances of 2.3, 3.2, 4.3, and 5.8 diameters from the back face of the wall to the center of the pile. After compaction, a 4 ft x 4 ft by 2 ft thick pile cap was poured around the test piles to produce a fixed-head condition. Roller bearings were placed between the base of the cap and plywood sheet on the ground surface to minimize base friction. Relative compaction of the soil adjacent to these piles was of about 100% relative to the Proctor standard test density.

Furthermore, strain gauges were placed along the reinforcements behind the MSE wall. This section of the wall had welded wire reinforcements. Figure 2-9 shows a diagram of the location of these strain gauges relative to the test pile caps and the MSE wall.

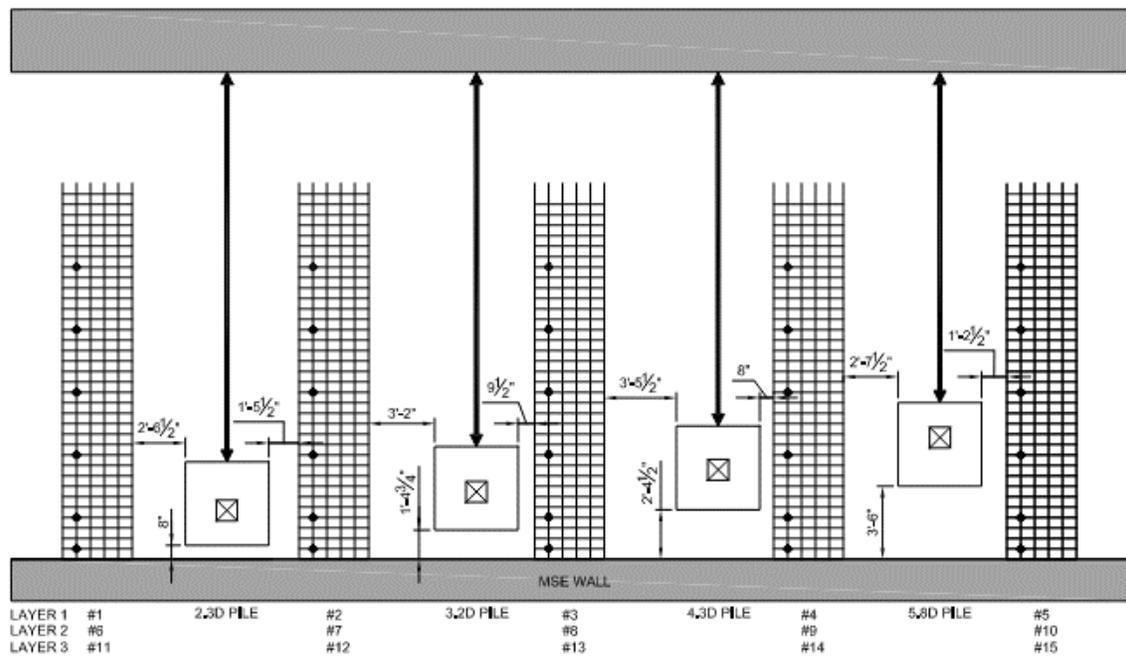


Figure 2-9. Location of the strain gauges in the welded wire reinforcements for fixed head test piles.

The predicted maximum tensile force was compared afterwards to the maximum observed tensile forces in the various reinforcements during lateral load testing of each pile. Figure 2-10 shows a comparison of the log of the measured maximum tensile force plus one relative to the log of computed maximum tensile force plus one. To compute the maximum tensile force in the reinforcement for welded wire equation 2-4 was used. It is important to notice that this equation was developed with free head conditions data. In addition, applied loads in this test were 150 to 300% greater than those applied to the free-head condition piles. Thus, this

significantly increased the measured tensile force leading to poor agreement with the predicted tensile force.

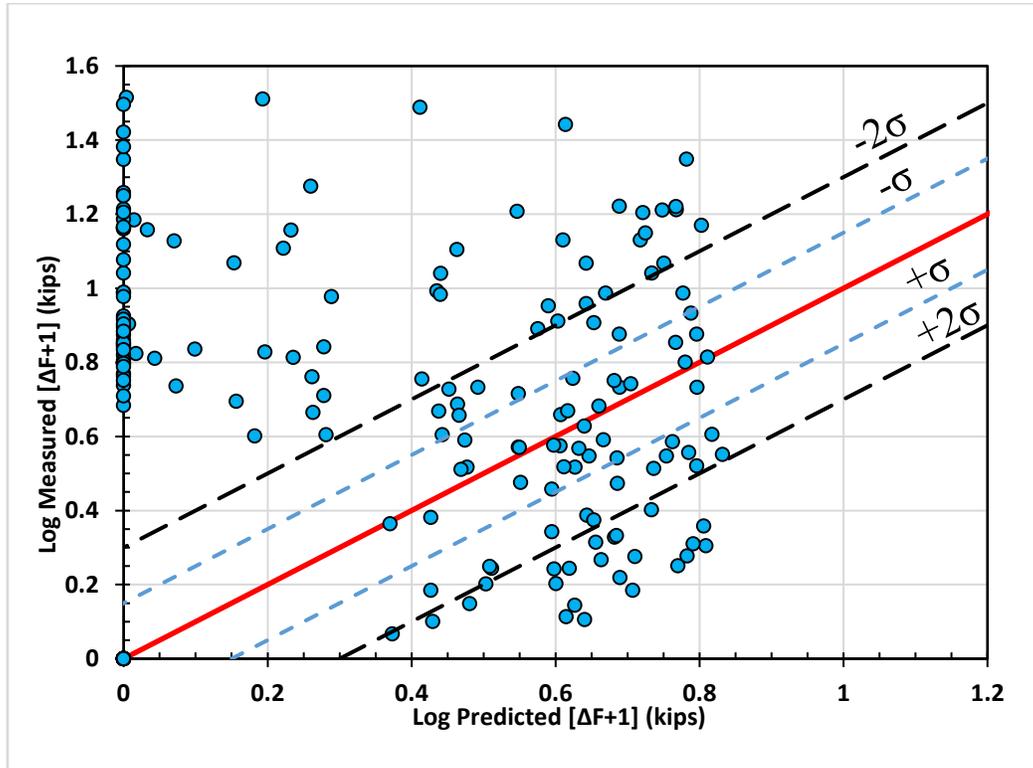


Figure 2-10. Statistical comparison of previously suggested equation for prediction of induced tensile force by soil reinforcements and the measured tensile force by welded wire reinforcements.

2.2.3 Using prediction equation on group piles

In addition, for phase 3 there were 3 piles of 12.75-inch diameter which were driven to 1.8, 2.8, and 3 diameters away from the face of the wall. This section of the wall had ribbed strip reinforcements and the relative compaction between the face of the wall and the piles was about 95% of the standard Proctor maximum density.

Strain gauges were placed in the ribbed strip reinforcement to measure the tensile force and the maximum was obtained from those to compare it with the calculated values with equation 2-

4. Figure 2-9 shows the location of the soil reinforcements respective to the location of the wall and the piles (Farnsworth, 2020).

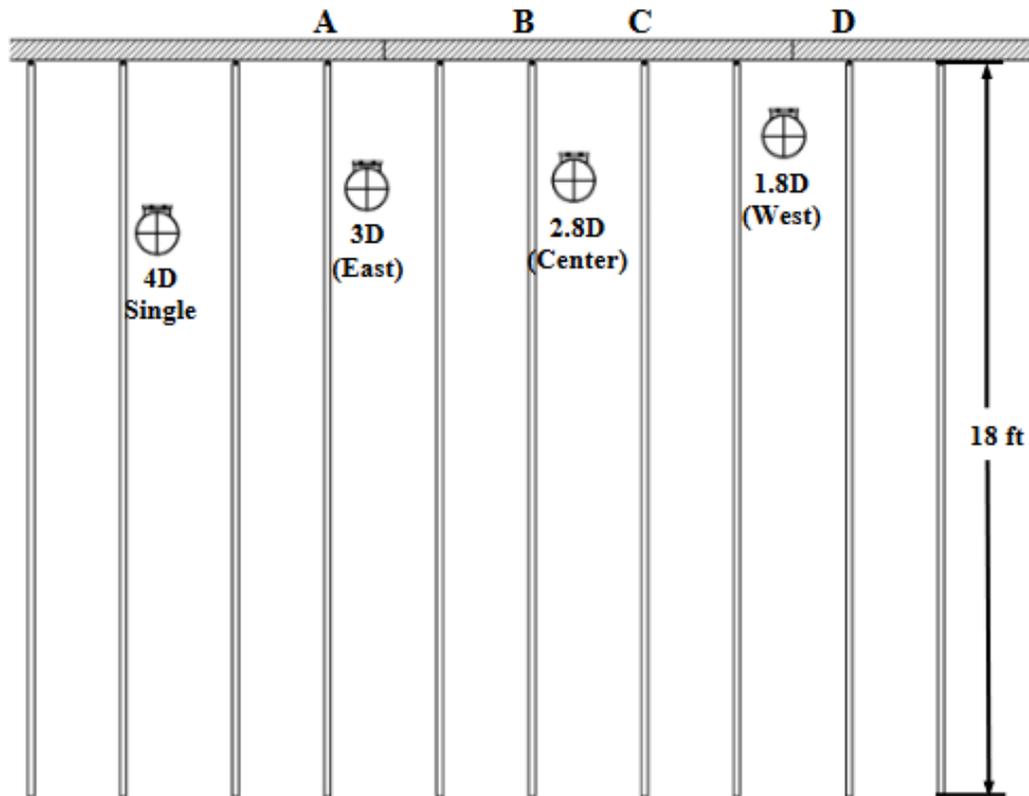


Figure 2-11. Location of ribbed strip reinforcements and the piles that were tested as a group (Farnsworth, 2020).

The maximum induced tensile force was calculated individually for each pile and it was superposed afterward to consider the effect of all the piles loaded as a group. Figure 2-12 shows the comparison of the log of the maximum measured tensile force in the ribbed strip reinforcements versus the log of the maximum predicted tensile force in ribbed strip reinforcements with equation 2-4.

The percentage of data points from this study within the one and two standard deviation boundaries are 63% and 85%, respectively, which is in good agreement with expectation for a normal distribution. However, using the superimposition approach with equation 2-4 tended to overestimate induced tensile forces for large pile head loads. Therefore, high lateral loads may be beyond the range of applicability for equation 2-4.

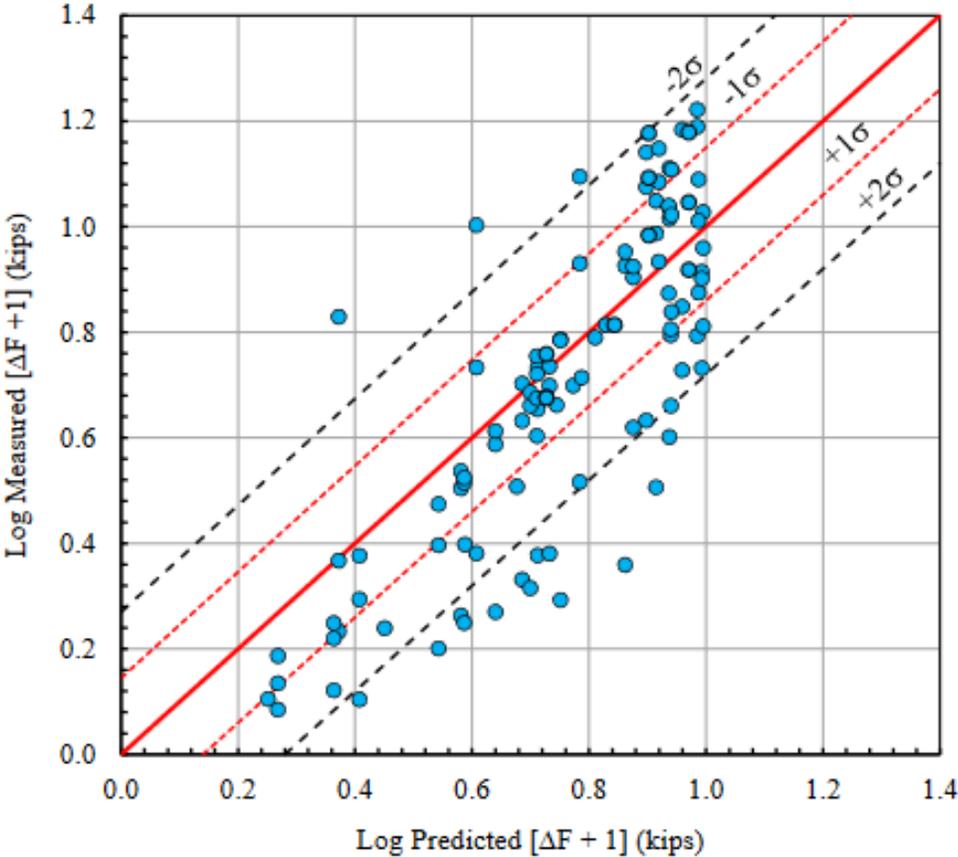


Figure 2-12. Measured versus predicted maximum tensile force in grouped pile soil reinforcements (Farnsworth, 2020).

3 STATISTICAL ANALYSIS

For practical purposes, explanatory or independent variables were normalized by dividing by certain values to make the input in the prediction empirical equations dimensionless. As was done previously, a logarithmic transformation was used for the measured induced force on the reinforcements because the forces appeared to be log-normally distributed. Transverse distances between the reinforcement and the center of the piles were divided by the diameter of the piles, while distances normal to the wall, from the back face of the wall to the center of the pile, were also divided by the pile diameter. The load measured by the load cell was divided by the axial yield force equal to the cross-sectional area of a 12.75-inch diameter pile times the yield strength. For example, for a 12.75” diameter pile, a constant value of yield force equal to 775 kips was calculated as the cross-sectional area of the 12.75” diameter steel pipe (13.59 in²) multiplied by the yield strength (f_y) of 57 ksi. Additionally, the vertical stress calculated at the depth of each reinforcement was divided by an atmospheric pressure value of 2109 psf. Table 3-1 summarizes the normalization values mentioned before.

Table 3-1: Values used to normalize the independent variables to make the prediction equations dimensionless.

Atmospheric pressure, p_a (psf)	Diameter of the pile, D (in)	Yield force, P_y (kips)
2100	12.75 – 24 (Variable)	775

As indicated previously, data was gathered from free-head lateral pile load tests conducted with approximately 12-inch diameter pipe, square and H piles adjacent to the MSE walls in phase 1 of the study previously collected by Luna (2016). In addition, we gathered data from the fixed-head tests on 12-inch H piles (Flores, 2019) as well as cyclic lateral free-head pile load

tests conducted on 12.75 inch diameter pipe piles (Wilson, 2019) adjacent to MSE walls in Phase 3. Finally, we collected data on lateral free-head pile load tests conducted by Wilson (2019) on 24-inch diameter pipe piles near the MSE walls Phase 3.

After gathering all the data from all the tests, the statistical software package JMP Pro 15 SAS was used to perform multi-variable regression analyses. Data from the induced tensile force on MSE walls with ribbed strip reinforcement and welded wire reinforcement were input separately as shown in Figure 3-1 and Figure 3-2.

	Tensile force +1 (Kips)	Log10[Tensile force +1 (Kips)]	Transverse distance ...	P/Py	Spacing (D)	σ/P_o	Relative compaction (%)
1	1.17	0.0681858617	2.86	0.02	2.3	0.67	100
2	1.75	0.2430380487	2.86	0.031	2.3	0.67	100
3	3.53	0.5477747054	2.86	0.048	2.3	0.67	100
4	5.4	0.7323937598	2.86	0.06	2.3	0.67	100
5	7.52	0.8762178406	2.86	0.069	2.3	0.67	100
6	9.7	0.9867717343	2.86	0.076	2.3	0.67	100
7	11.69	1.0678145112	2.86	0.082	2.3	0.67	100
8	13.5	1.1303337685	2.86	0.087	2.3	0.67	100
9	1.41	0.1492191127	1.84	0.02	2.3	0.52	100
10	1.75	0.2430380487	1.84	0.031	2.3	0.52	100

Figure 3-1: Variables input in JMP for MSE wall with welded-wire reinforcement.

	Tensile force+1 (kips)	Log10[Tensile force+1 (kips)]	Transverse distance...	P/Py	Spacing (D)	σ/P_o	Diameter /12.75	Relative compaction %
1	3.91	0.5921767574	0.66	0.075	2	0.52	1.88	95.04
2	7.11	0.8518696007	0.66	0.103	2	0.52	1.88	95.04
3	11.8	1.0718820073	0.66	0.15	2	0.52	1.88	95.04
4	14.84	1.1714339009	0.66	0.181	2	0.52	1.88	95.04
5	16.98	1.2299376859	0.66	0.207	2	0.52	1.88	95.04
6	18.58	1.2690457097	0.66	0.223	2	0.52	1.88	95.04
7	2.97	0.4727564493	1.81	0.075	2	0.52	1.88	95.04
8	4.35	0.638489257	1.81	0.103	2	0.52	1.88	95.04
9	7.14	0.8536982118	1.81	0.15	2	0.52	1.88	95.04

Figure 3-2: Variables input in JMP for MSE wall with ribbed strip reinforcement.

For practical purposes, the prediction equation was required to have less than five terms involved. Therefore, for the variable selection, a forward stepwise regression was used with a Bayesian information criterion as a stop rule. For the terms evaluated, single explanatory variables, interaction terms and quadratic terms were introduced into the stepwise regression.

Afterwards, the terms were reduced according to p-value from the highest to the lowest ones until having five terms. A lower p-value means that the term has a higher statistical significance in the empirical equation.

Furthermore, having all the terms of higher significance, a multiple linear regression was developed using JMP Pro 15. Equation 3-1 is the general multiple linear regression equation; coefficients were calculated with the least squares method to approximate its solution to the minimum sum of squared residuals.

$$\text{Log}(y + 1) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_1x_2 + \beta_4x_1^2 \dots + \beta_nx_n + +\beta_mx_nx_m + \beta_px_n^2.. \quad (3-1)$$

Where:

β_i : Parameter's coefficient

x_2 : Explanatory variable

y : Dependent variable

Parameter's estimates were obtained from JMP 15 Pro with an analysis of variance. The p-value was evaluated for every parameter to determine its statistical significance. Figure 3-3 shows the output in JMP 15 pro and the statistics of each specific parameter obtained from the software.

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.7170101	0.02295	31.24	<.0001*
P/Py	9.2298705	0.195253	47.27	<.0001*
(P/Py-0.04354)*(P/Py-0.04354)	-51.40565	2.540287	-20.24	<.0001*
Spacing (D)	-0.038742	0.003422	-11.32	<.0001*
Transverse distance (D)	-0.073717	0.003904	-18.88	<.0001*
σ/Po	-0.37122	0.026394	-14.06	<.0001*

Figure 3-3: Parameter estimates obtained from JMP 15 pro

There are some conditions that a statistical model does not consider when making a prediction equation. One condition considered is that when the applied force on the pile was zero

the induced force on the reinforcement was zero as well. Additionally, when the prediction equation calculated a negative induced force, the force was considered to be zero. Measured vs predicted charts were developed for each equation to help evaluate its accuracy visually. In addition, the coefficient of correlation, or R^2 value, was determined for each relationship to provide a more quantitative estimate of each equation. The correlation coefficient gives an estimate of the fraction of the variability that the regression equation can explain. For example, a correlation coefficient of 0.75 indicates that 75% of the variability in the data is explained by the prediction equation.

3.1 Correlations for Ribbed Strip Reinforcements

3.1.1 Predictive equation for 12-inch diameter piles for phase 1 and 2

As noted previously, explanatory variables from the previously developed equations for ribbed strip reinforcement (Luna, 2016) were divided by the constant values in Table 3-1 to make the input of the values dimensionless. This artifice would ensure that the units of the input would not matter if the units of the numerator and denominator values were consistent.

Additionally, values that were calculated as negative were assumed to be zero; furthermore, the tensile force was assumed to be zero when the applied load at the pile head was zero. The prediction equation kept the same explanatory variables as with the previous equation and vertical stress was not a variable with statistical significance as was the case for the previous equation developed by Luna (2016).

The maximum tensile force (ΔF) induced in the reinforcements by an applied pile head load is given by the prediction equation,

$$\Delta F(kips) = 10^{[0.134 + 22.09 \frac{P}{P_y} - 132.8 \left(\frac{P}{P_y}\right)^2 - 9.7 \times 10^{-3} \frac{T}{D} - 1.614 \left(\frac{T}{D}\right) \left(\frac{P}{P_y}\right) - 0.031 \frac{S}{D}] - 1} \quad (3-$$

2)

where:

P is the pile head load in units of force

P_y is the pile yield force under axial compression in units of force (775 kips)

T is the transverse distance between the center of the pile and the center of the reinforcing strip
in units of length

S is the distance from the back face of the wall to the center of the pile in units of length, and

D is the diameter of the pile in units of length.

Figure 3-4 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3.2. There are 942 points represented in this plot and the R^2 value is 0.72. The standard deviation of the $\log(1+\Delta F)$ is 0.14. The 1:1 red continuous line represents calculated values that have perfect agreement with the measured values, the blue dashed lines represents values that are one standard deviation away from the perfect agreement and the black dashed lines represents values that are two standard deviations away from the perfect agreement line. This R^2 value is comparable to that developed by Luna (2016) based on input parameters that were not normalized. For low induced tensile force, the equation seems to overpredict given that several points are below the red 1:1 line, but afterwards there appears to be less bias and more data points lie closer to the perfect agreement line.

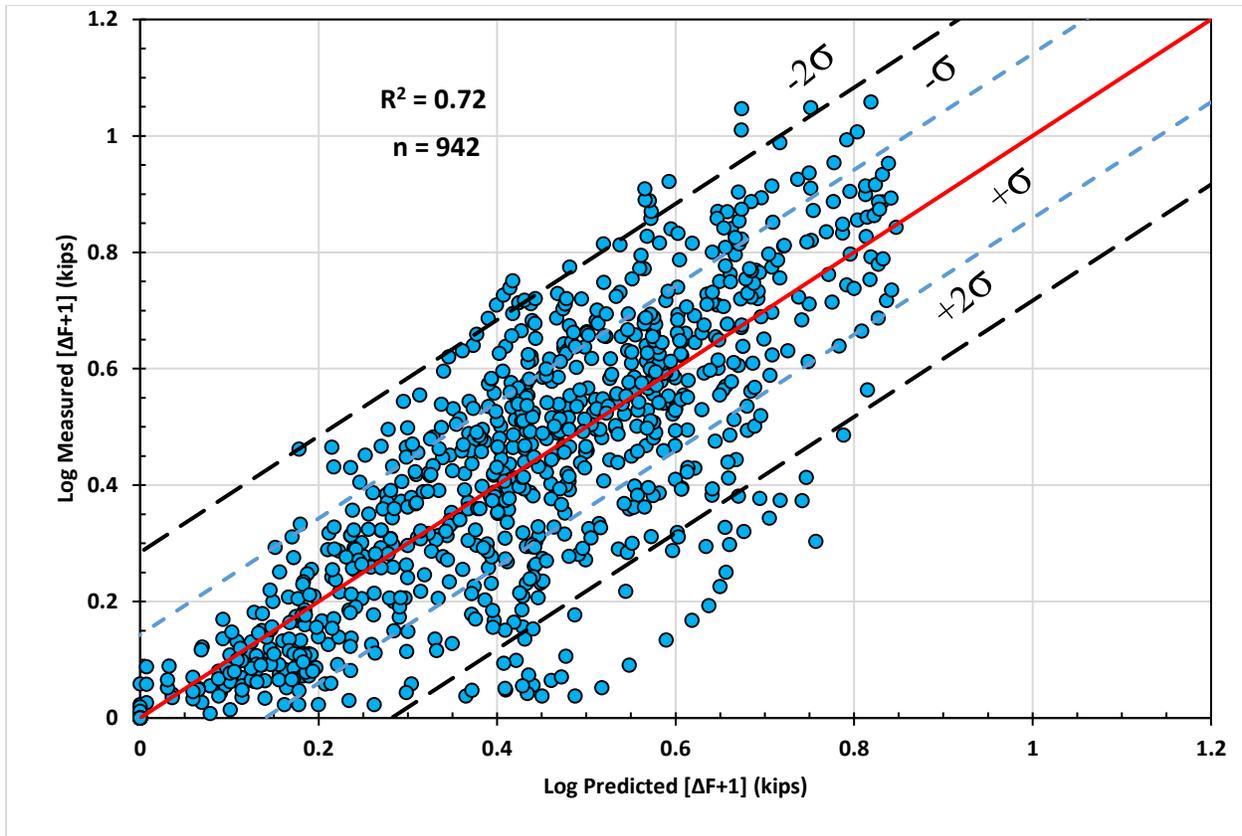


Figure 3-4: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one comparison for Phase 1 and 2 on ribbed strip reinforcement using dimensionless variables.

3.1.2 Predictive equation for 24-inch diameter piles

Eq. 3-2, previously developed for phase 1 and 2 testing, has certain limitations. For example, it does not consider piles with other diameters than about 12-inches, the soil from the data gathered in phase 1 and phase 2 was compacted to about 90% relative compaction relative to the standard Proctor test, and pile head loads limited to about 56 kips.

During phase 3, researchers (Wilson, 2020), studied 24-inch diameter pipe piles laterally loaded behind MSE walls reinforced with ribbed strip reinforcements. Induced tensile force data from this test was used with Eq. 3-2 as shown in Figure 2-8 and the measured maximum tensile forces in the reinforcements for these tests were considerably higher than those predicted using Eq. 3-2 because of the larger pile diameter and the larger pile head loads. Therefore, another

regression equation was developed to predict the measured maximum tensile force for this set of load test data.

Using the same methodology described at the beginning of this chapter an empirical prediction equation was developed with data from the full-scale test reported by Wilson (2018). Variables were also divided by the values in Table 3-1 to make the input of the equation dimensionless. In addition, negative values, and values where the pile head load of the pile was zero were automatically assumed to be zero. Compaction was not considered as an independent explanatory variable for this case since all the data for this test had a relative compaction of 95% based on the standard Proctor test.

The maximum tensile force (ΔF) induced in the reinforcements by an applied pile head load is given by the prediction equation,

$$\Delta F(kips) = 10^{0.65} \left[6.87 \frac{P}{P_y} - 11.92 \left(\frac{P}{P_y} \right)^2 - 4.92 \left(\frac{\sigma_v}{p_a} \right)^2 + 4.76 \frac{\sigma_v}{p_a} - 0.125 \frac{S}{D} - 0.65 \right] - 1 \dots \dots \dots (3-3)$$

Where:

P is the pile head load in units of force

P_y is the yield force of a 12.75” diameter pile under compression in units of force, (775 kips)

S is the distance from the back face of the MSE wall to the center of the pile in units of length

D is the diameter of the pile in units of length.

σ_v is the vertical stress of the soil at the depth of the reinforcing strip in units of pressure, and

p_a is the atmospheric pressure (a constant value) in units of pressure.

Equation 3-3 was developed exclusively for 24-inch diameter piles, as mentioned before, with pile head loads in the range from 58 kips to 212 kips. These are some of the limitations of this

equation, however, hereafter regression equations will be described using all available pile test data.

In contrast to Equation 3-2, developed for the 12.75 inch piles, the vertical effective stress had statistical significance and was included in the equation. In contrast, one variable that did not have statistical significance for this test was the transverse distance between the reinforcing strip and the center of the pile. This might be due to the tested piles being too close to each other, thus, making the backfill failure planes overlap (Wilson, 2020). In addition, the horizontal distance from the pile to the reinforcements were all less than 1.8 pile diameters from the center of the pile. This may have provided insufficient information by which to see the effect of transverse distance.

Figure 3-5 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-3. For this case, 136 data points were collected, the R^2 is 0.69, and the standard deviation of the $\log(1+\Delta F)$ is 0.14. A review of the data does not suggest any apparent bias in the predictions. Figure 3-5 illustrates that the data is very well distributed for the different measurements of induced tensile force and that there are no induced tensile force data points less than 1.37 kips [i.e., $\text{Log}(\Delta F+1) = 0.37$] due to the high loads involved in this test. Furthermore, Equation 3-3 can predict tensile force of up to 14.8 kips [i.e., $\text{Log}(\Delta F+1) = 1.19$] with reasonable accuracy (typically within one standard deviation).

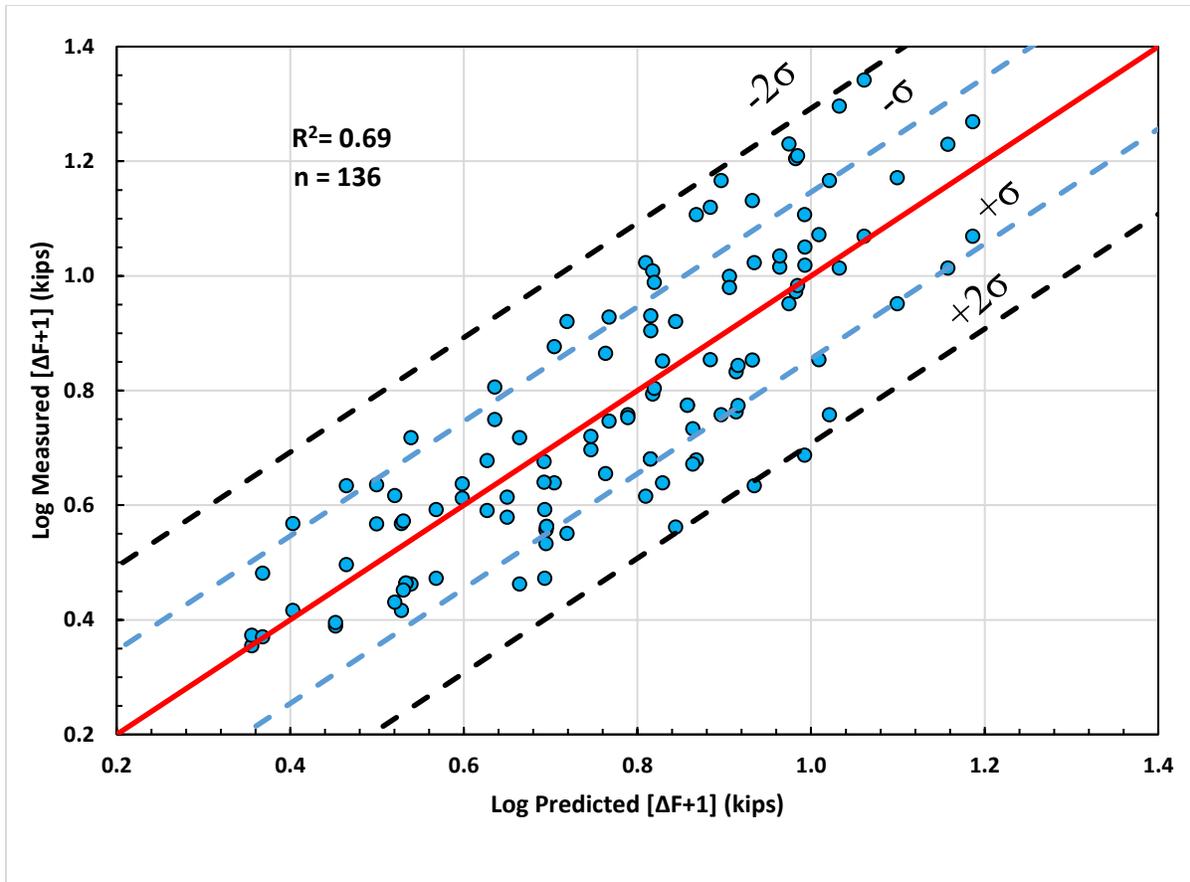


Figure 3-5: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one in ribbed strip reinforcement comparison for laterally loaded 24” diameter piles behind MSE wall.

3.1.3 Predictive equations for all pile tests

Although the predictive equations for the 12-inch (Equation 3-2) and 24-inch diameter (Equation 3-3) pile tests both provide reasonable estimates of the measured maximum tensile force in the reinforcements, it would be desirable to develop one equation that could account for the entire range of pile diameters and geometric variables. To investigate this possibility, test data from the 24-inch and 12-inch diameter piles test was combined together, and a new regression equation was developed for the entire data set from phases 1, 2 and 3.

Given that the phase 3 testing included several changes, the diameter of the pile and the relative compaction became explanatory variables of interest to study. To keep the pattern of

producing an equation where the input variables are dimensionless, the diameter variable was divided by a constant value of 12.75-inches, because the diameter of most of the piles in the dataset was close to this value.

The maximum tensile force (ΔF) induced in the reinforcements by an applied pile head load (P) is given by the prediction equation,

$$\Delta F(kips) = 10^{[17.83 \frac{P}{P_y} + 0.233 \frac{D}{D_o} - 3.83 \times 10^{-2} \frac{S}{D} - 5.8 \times 10^{-2} \left(\frac{T}{D}\right) - 8.22 \left(\frac{T}{D}\right) \left(\frac{P}{P_y}\right) - 0.12]} - 1$$

(3-4)

Where:

P_y is the pile's yield force under compression in units of force (775 kips)

T is the transverse distance between the center of the pile and the reinforcing strip in units of length

S is the distance from the back face of the wall to the center of the pile in units of length

D is the diameter of the pile in units of length.

D_o is a constant diameter of 12.75" in consistent units of length consistent with D

For this equation, negative predicted values of tensile force are assumed to be zero and tensile force was automatically assumed to be zero when the pile head load was zero.

Equation 3-4 is based on 1078 data points which are all from full-scale tests. The R^2 for this prediction equation is 0.72 and the standard deviation of the $\log(1+\Delta F)$ is 0.16. It is noted that the relative compaction was not one of the final explanatory variables, however, this does not mean that it does not have statistical significance. It simply indicates that the other variables had greater statistical significance. As the selection of variables was being performed, the R^2 of the multiple linear regression was 0.75 and involved three terms that included relative compaction (R_c). However, at this point, the regression equation had 8 terms, making it more

difficult to apply in practice. Therefore, we decided to remove the relative compaction terms for the sake of simplicity. Additionally, for this case the vertical effective stress was not one of the variables with statistical significance as was the case for the equation based only on the 12-inch diameter pile tests.

Figure 3-6 shows the logarithm of measured maximum induced tensile force plus one in vs the logarithm of the calculated maximum induced tensile force (in kips) plus one using equation 3-4. Square green data points are those that were added from phase 3 testing with the 24-inch diameter piles and the blue circles were all data points from phase 1 and phase 2 with the 12-inch diameter piles. The agreement between the measured and computed data points is reasonably good and there does not appear to be any obvious bias. Nevertheless, the prediction of the tensile force for the 24-inch diameter piles appears to be a little less accurate than that for the 12-inch piles.

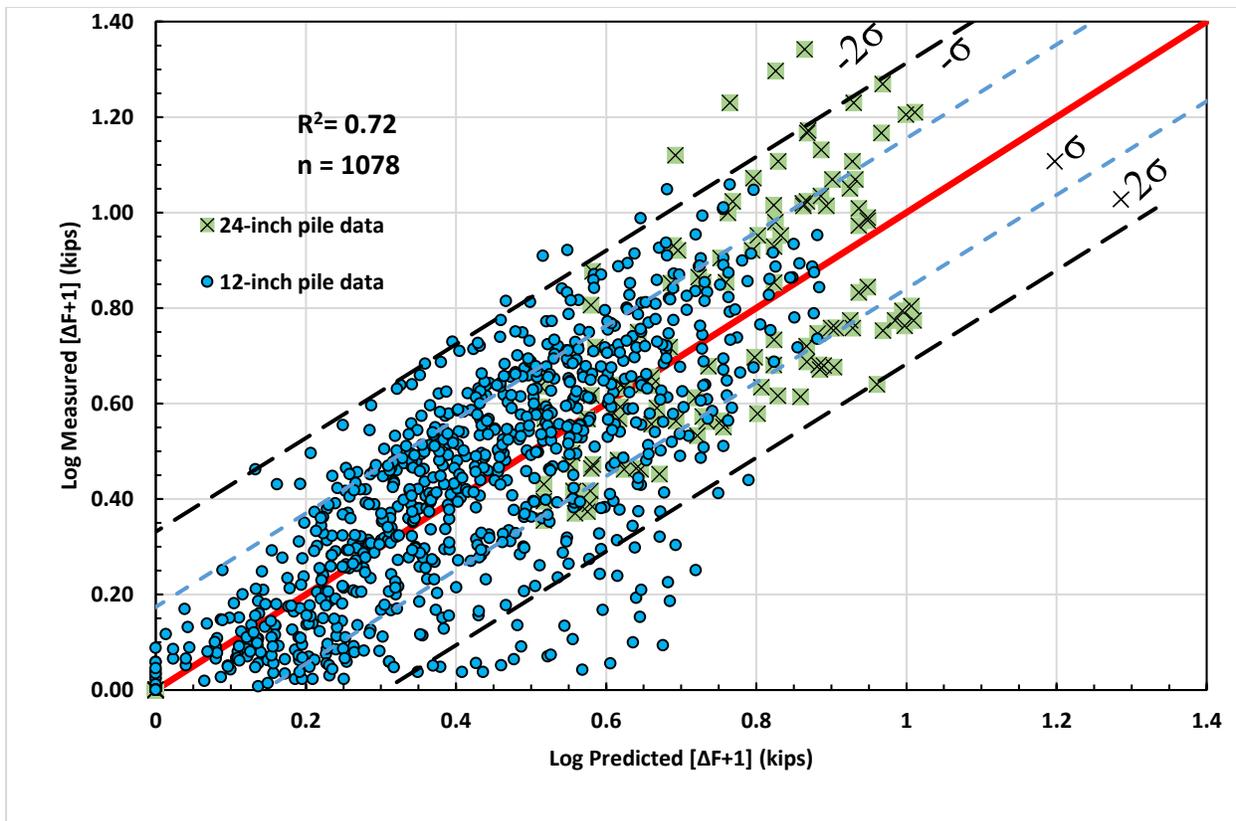


Figure 3-6: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one in ribbed strip reinforcement comparison for all diameters.

A comparison of the Phase 1 & 2 equation with the Phase 3 equation is provided with Figure 3-7. The equations were tested keeping all the variables constant and increasing the load. The reason why the load was chosen as the variable factor is because it is the explanatory variable that has the most statistical significance. Constant values assumed for this comparison were a transverse distance of the center of the pile to the reinforcing strip of $0.66D$, spacing between the back face of the wall to the center of the pile of $2D$, and a diameter ratio of 1 (i.e., $D = 12.75''$).

Round data points are maximum induced tensile force calculated with the phase 1, 2 and 3 prediction equation and triangular data points were calculated with the phase 1 & 2 prediction equation.

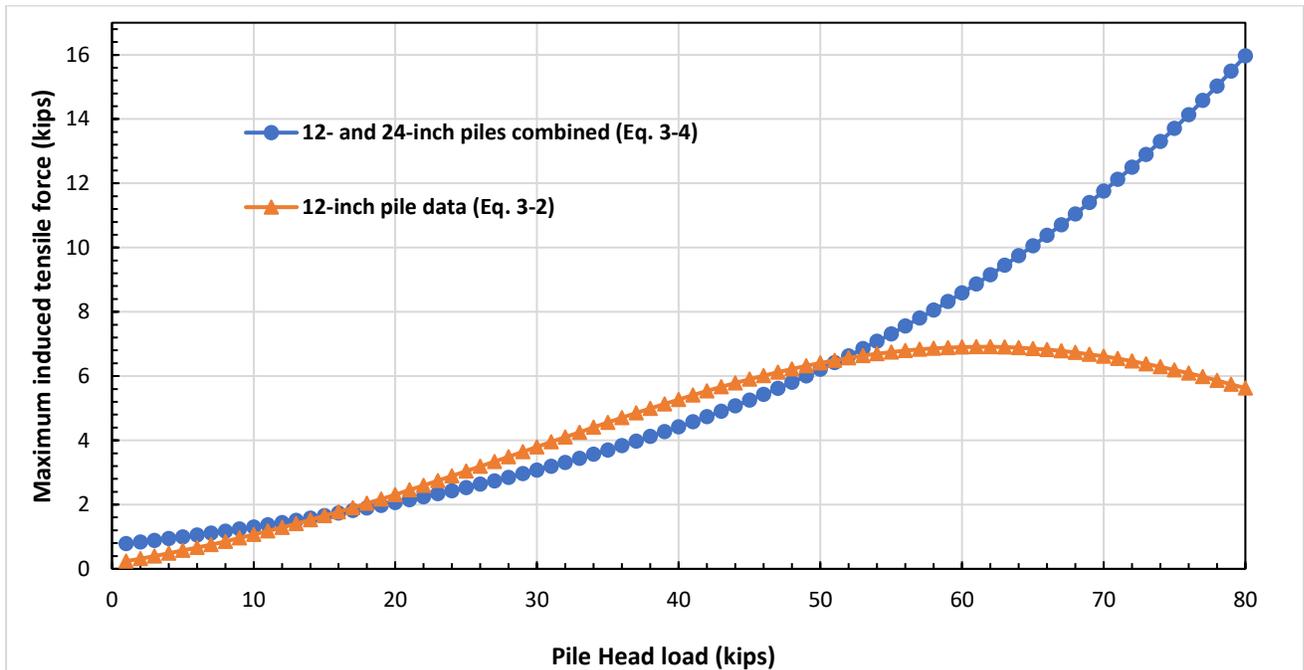


Figure 3-7: Comparison of the maximum induced tensile force calculated with Phase 3 and Phase 1 & 2 prediction equations for a laterally loaded 12.75-inch diameter pile.

The predicted values for the combined data set (Equation 3-4) are very similar to the predicted values for the 12-inch piles (Equation 3-2) up to a pile head load of about 52 kips. However, at higher pile head loads, Equation 3-2 begins to significantly underestimate the computed force relative to Equation 3-4. The equation for the combined data set provides a better estimate of the maximum induced tensile force for higher head pile loads and it can predict higher induced tensile force than the equation developed for the 12-inch piles where the pile head loads were lower as can be observed in Figure 3-7.

Another variation of the 24-inch pile testing that was included with the higher loads was the variation with the pile diameter. Thus, we have also made comparisons between Equations 3-2 and 3-4 keeping all the values constant but increasing the load. However, this time we have used a pile diameter ratio of 1.88 (i.e., $D = 24$ -inches). All the other values were the same as with the previous comparison. Figure 3-8 shows the comparison of the maximum induced tensile force with increasing pile head load for a 24-inch pile diameter.

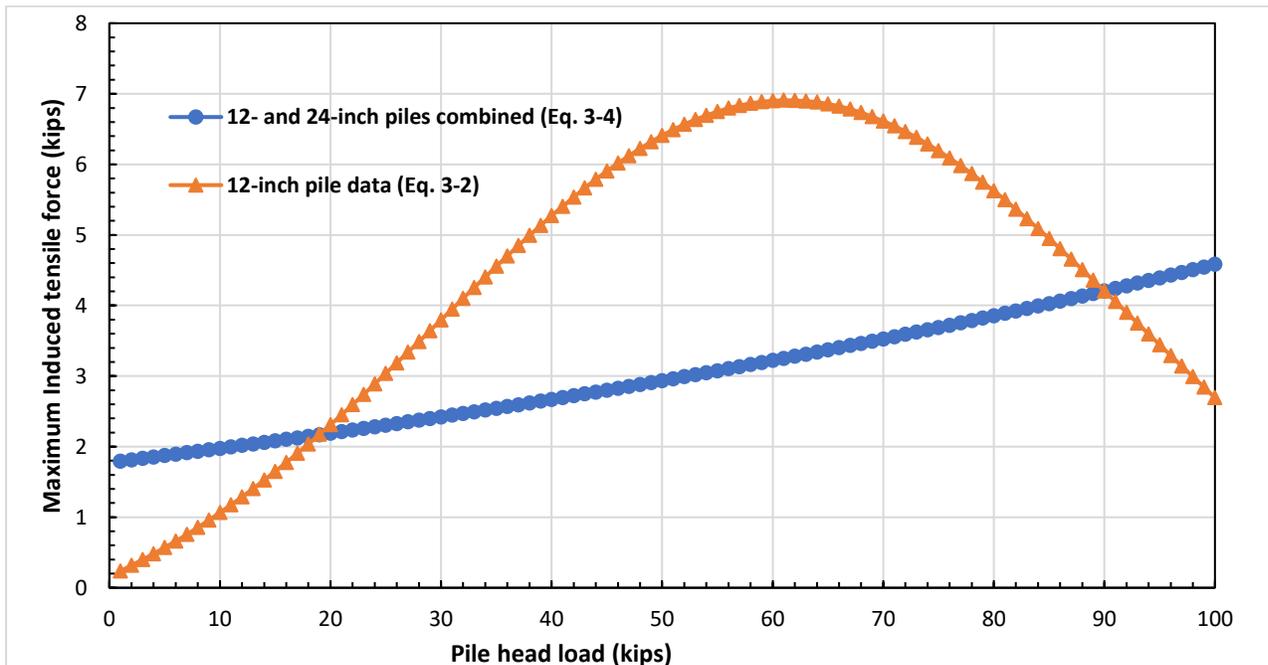


Figure 3-8: Comparison of the maximum induced tensile force calculated with Phase 3 and Phase 1 & 2 prediction equations for a laterally loaded 24-inch diameter pile.

One effect that can be observed from the comparison of the equations is that the combined equation (Equation 3-4) computes smaller maximum induced tensile forces and there is a notable difference between the predicted values for both equations. The reason that the 24-inch diameter pile causes less induced tensile force on ribbed strip reinforcements is because a laterally loaded 24-inch diameter pile deflects less than a 12.75-inch diameter pile for the same load. It makes the soil, and its reinforcing elements receive less strain from the stresses induced due to the laterally loaded pile. This comparison shows that the equation based on the 12-inch pile tests is not very suitable for laterally loaded large diameter piles.

Additionally, Equation 3-4 based on all the test data seems to over-predict tensile force for pile head loads below 20 kips. The reason for this is that the data for large diameter piles has pile head loads higher than about 60 kips and our multi-linear regression model does not include data for loads lower than that for the larger diameter piles.

3.1.4 Group piles

Another test that was performed for phase 3 included piles laterally loaded as a group behind an MSE wall (Farnsworth, 2020). From Figure 2-12 it was observed that superposing the tensile force on the reinforcing strips caused by the laterally loaded piles provides accurate calculations. In addition, the original equation based on the 12-inch pile testing in phase 1 & 2 equation was tested, and it predicted maximum tensile force relatively well.

Figure 3-9 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using superposed data from the group piles test with Equation 3-4.

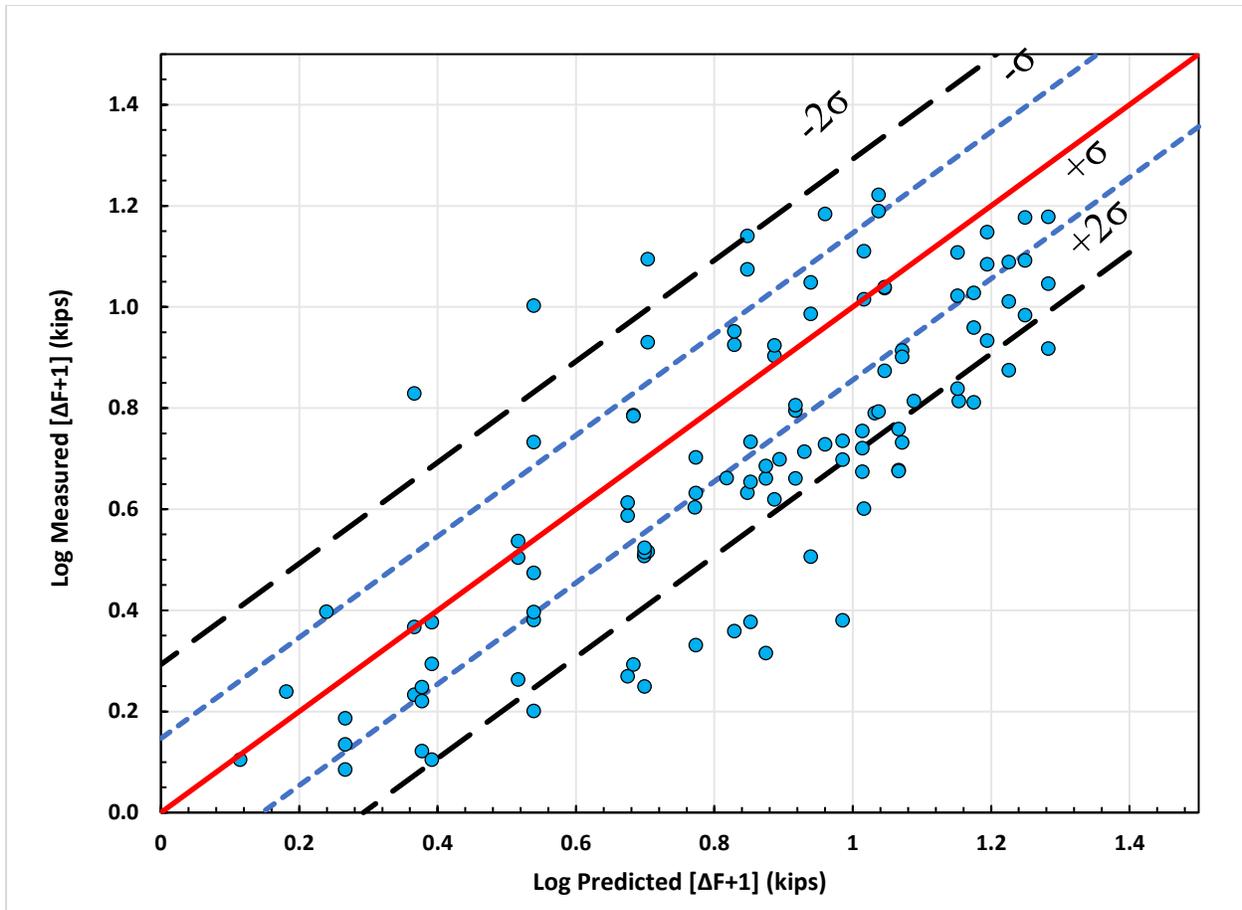


Figure 3-9: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one comparison using data from group pile test with Equation 3-4.

There are 39 data points that lie within the one standard deviation region which is 39 percent of the data, and 86 data points that fall within two standard deviation region which is 78.9 percent. This shows that the equation is not as accurate as would be expected for a normal distribution. In contrast, the data points seem to be spread out more uniformly. For ideal results there should be 68.27% data points within the one standard deviation region and 95.45 percent data points within the two standard deviation region.

Equation 3-3 which was developed with the large diameter piles test data, was also tested with this data from the group pile test given that the soil conditions were very similar. Figure 3-

10 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-3.

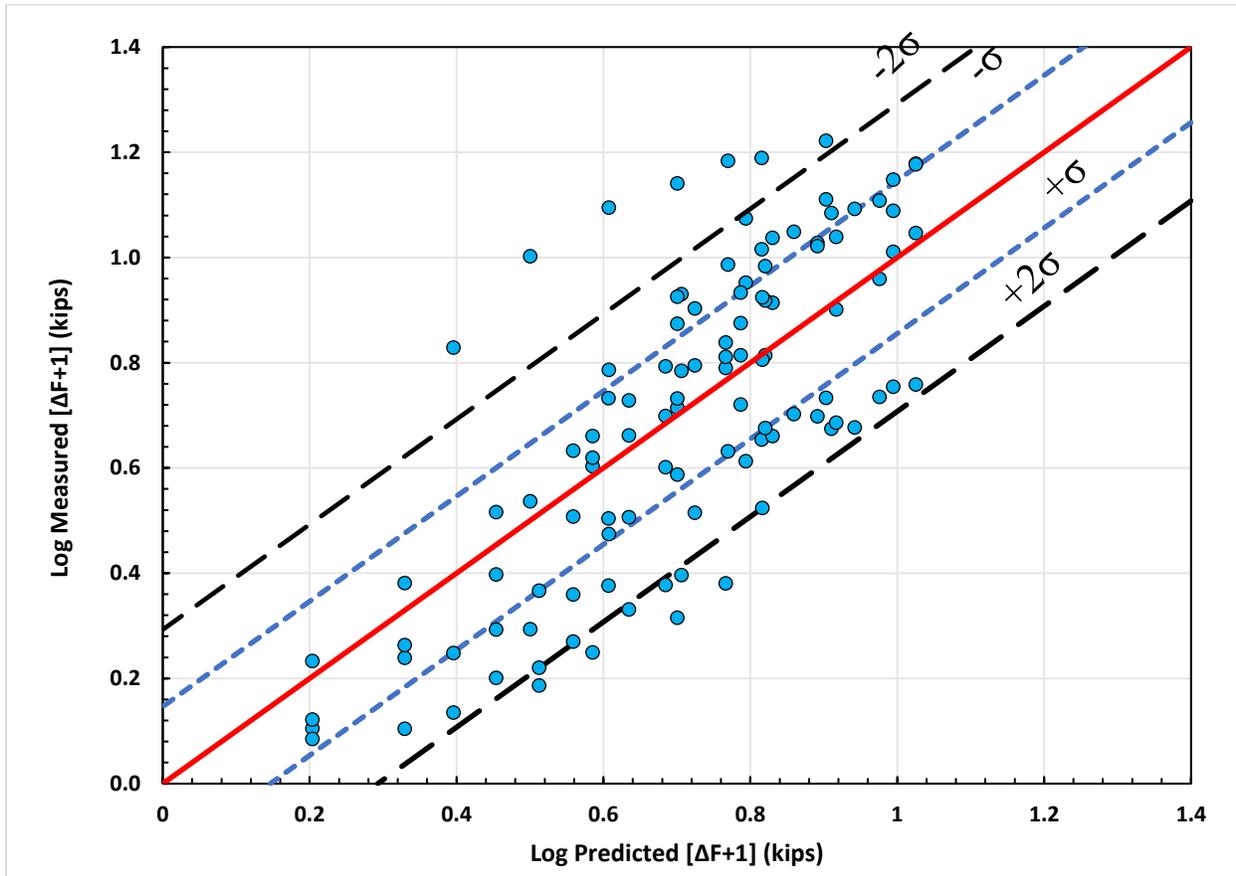


Figure 3-10: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one comparison using data from group pile test with large diameter piles prediction equation.

There are 50 data points within the one standard deviation region which is 45.9 percent of the data and 95 data points within the two standard deviation region which is 87.2 percent. This seems to be a more suitable equation for this case. The reason behind these results is that the soil conditions in this test were very similar. Both tests were performed on the same MSE wall at the same level of compaction which led to better agreement despite using the different pile diameter.

3.2 Welded Wire Reinforcement

3.2.1 Predictive equation for 12-inch piles (Phase 1 and 2 testing)

Variables from the previously developed data set for lateral pile load tests next to the MSE wall with welded wire reinforcements (Luna, 2016) were also normalized by dividing by the constant values in Table 3-1 to make the input of the equation dimensionless. In addition, in the process of selecting variables for this equation, a combination of explanatory variables was found that improved the accuracy of this prediction equation. Furthermore, if the applied load at the pile head was zero, the induced tensile force in the reinforcement was automatically assumed to be zero. Likewise, values that were calculated as negative were assumed to be zero. This approach increased the R² value to 0.74. The standard deviation of the log(1+ΔF) is 0.14.

Equation 3-4 is the prediction equation developed with the conditions mentioned before. Figure 3-11 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-4. The 1:1 red continuous line represents calculated values that have perfect agreement with the measured values, the blue dashed line represents values that are one standard deviation away from the perfect agreement and the black dashed line represents values that are two standard deviations away from the perfect agreement line.

The maximum tensile force (ΔF) induced in the reinforcements by an applied pile head load (P) is given by the prediction equation,

$$\Delta F(kips) = 10^{[13.67 \frac{P}{P_y} - 51.04 \left(\frac{P}{P_y}\right)^2 - 0.074 \frac{T}{D} + 0.37 \frac{\sigma_v}{p_a} - 0.039 \frac{S}{D} + 0.62]} - 1 \dots\dots\dots (3-4)$$

Where:

P is the pile head load in units of force

P_y is the yield force of a 12.75” diameter pile under compression in units of force (775 kips)

S is the distance from the back face of the MSE wall to the center of the pile in units of length

D is the diameter of the pile in units of length.

σ_v is the vertical stress of the soil at the depth of the reinforcing strip in units of pressure

p_a is the atmospheric pressure (a constant value) in units of pressure, and

T is the transverse distance from the center of the pile to the reinforcing grid.

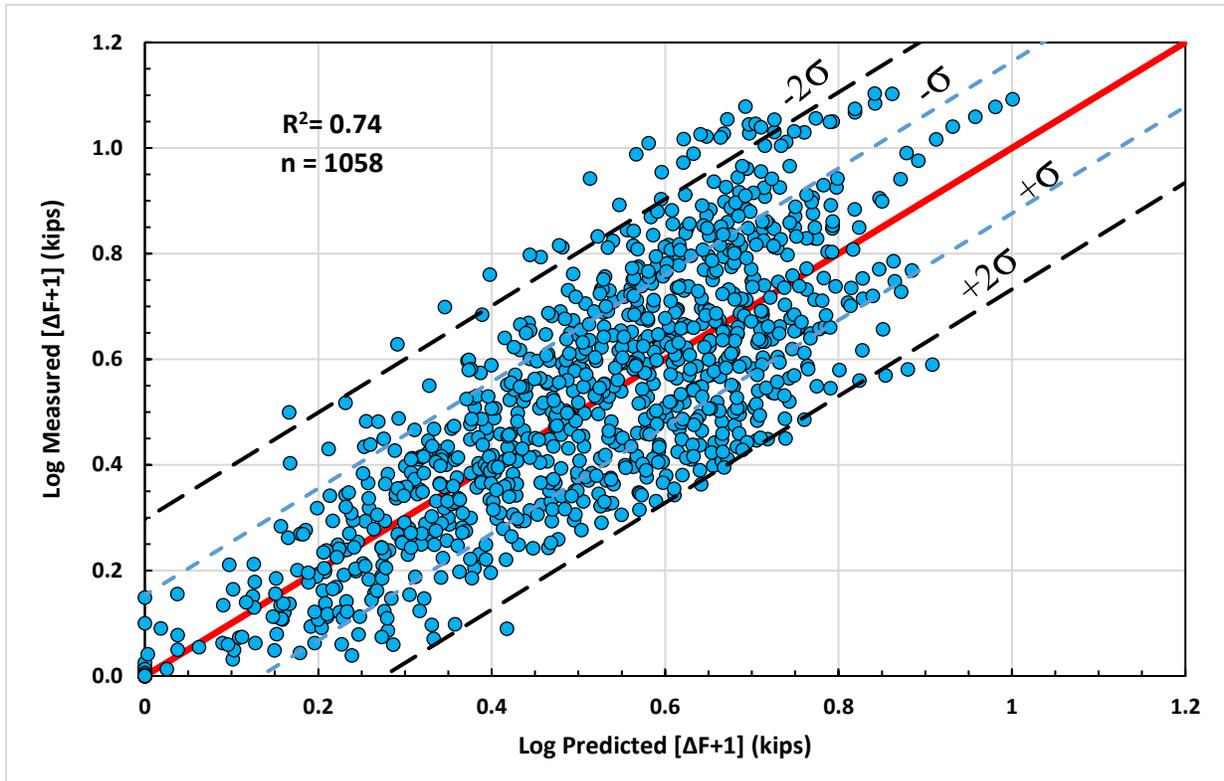


Figure 3-11: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one comparison for Phase 1 and 2 on welded wire using the dimensionless input variables.

Equation 3-4 was tested with the data obtained from the test where piles with fixed head conditions were laterally loaded behind an MSE wall that was reinforced with welded wire. Figure 3-12 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-4.

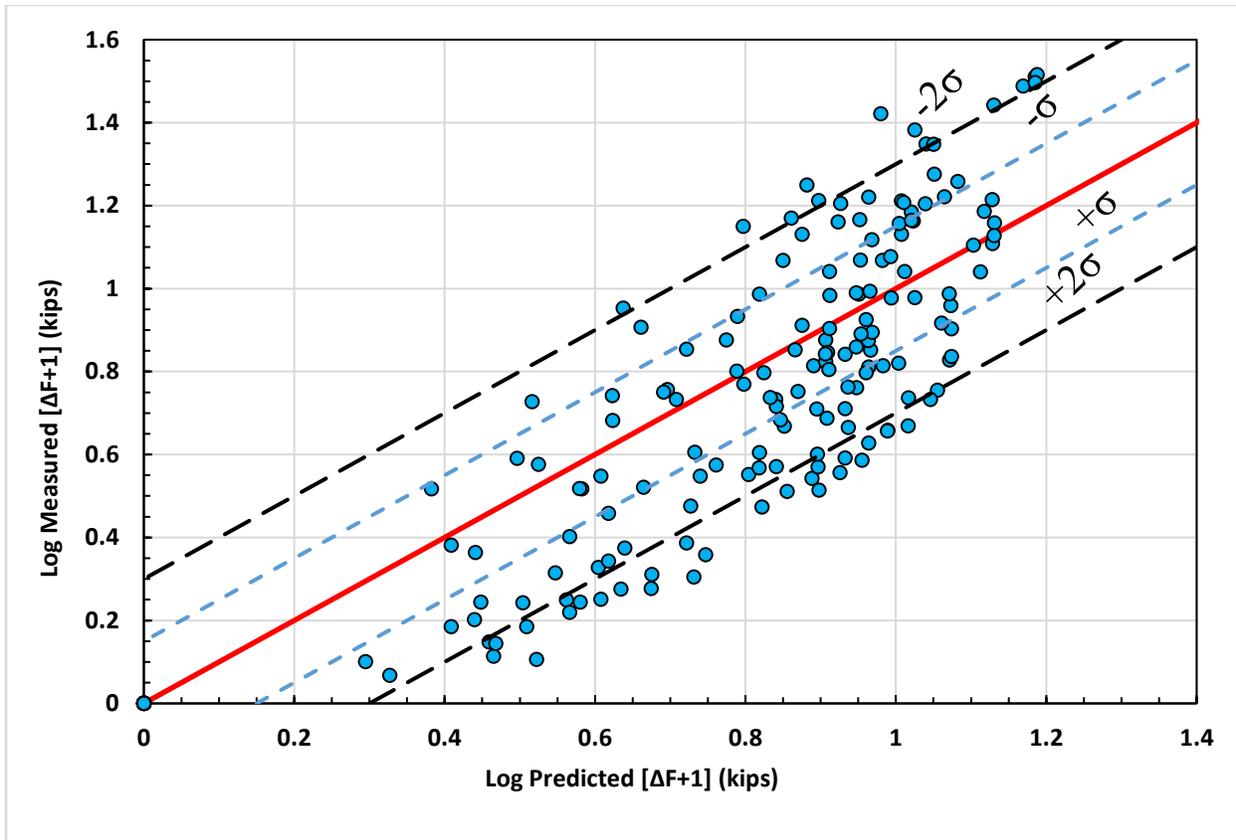


Figure 3-12: Logarithm of measured induced tensile force plus one for data obtained from fixed head conditions test vs logarithm of predicted induced tensile force plus one using Equation 3-4.

Most of the tensile forces seem to be over-predicted since most of the points lie below the red line, which means that the predicted values are higher than the measured values. However, for a calculated tensile force of around 9 kips and higher [$(\log\Delta F+1) > 1$] there are several under predicted values where measured force substantially higher than predicted.

In addition, only 34% of the data points lie within the one standard deviation region and 78% of the data points lie within the two standard deviations region. Therefore, this equation does not have the same degree of accuracy for fixed head conditions relative to free-head conditions.

3.2.2 Predictive equation for piles with fixed head condition

As noted previously, a fixed head condition was one of the limitations that was not considered in phase 1 and 2 testing with the MSE wall reinforced with welded wire. As shown in Figure 3-12 the data for the fixed head tests shows more scatter than desired for the equation developed with dimensionless inputs for welded wire, however, making an equation with just data from fixed head conditions could improve the accuracy of calculations for that condition. One unique condition for this test was that the soil was compacted to about a 100% relative compaction for a standard Proctor test.

Based on the regression analysis, the maximum tensile force (ΔF) induced in the reinforcements by an applied pile head load is given by the prediction equation,

$$\Delta F(kips) = 10^{[5.57 \frac{P}{P_y} - 0.076 \frac{S}{D} - 0.425 \frac{T}{D} - 2.81 \frac{\sigma_v}{p_a} + 0.65 \left(\frac{\sigma_v}{p_a}\right) \left(\frac{T}{D}\right) + 2.21]} - 1 \dots \dots \dots (3-5)$$

Where:

P is the pile head load in units of force

P_y is the yield force of a 12.75” diameter pile under compression in units of force (775 kips)

S is the distance from the back face of the wall to the center of the pile in units of length

D is the diameter of the pile in units of length.

σ_v is the vertical stress of the soil at the depth of the reinforcing strip in units of pressure

p_a is the atmospheric pressure (a constant value) in units of pressure.

T is the transverse distance from the center of the pile to the reinforcing grid

Performing the stepwise regression to select explanatory variables, using the Bayesian information criterion stop rule, there were 11 terms in the equation and the R^2 was 0.9, which shows that the computed tensile force correlated very well; however, this equation was judged to be too complicated. Reducing the equation to five terms only decreased the R^2 value to 0.85,

which is a relatively small decrease considering the increased simplicity of the equation. The standard deviation of the $\log(1+\Delta F)$ is 0.15. Figure 3-13 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-5. There are 194 data points represented in this plot. There are no apparent biases or skews in the data and the equation appears to provide a robust approach for predicting induced tensile for the fixed head condition test condition.

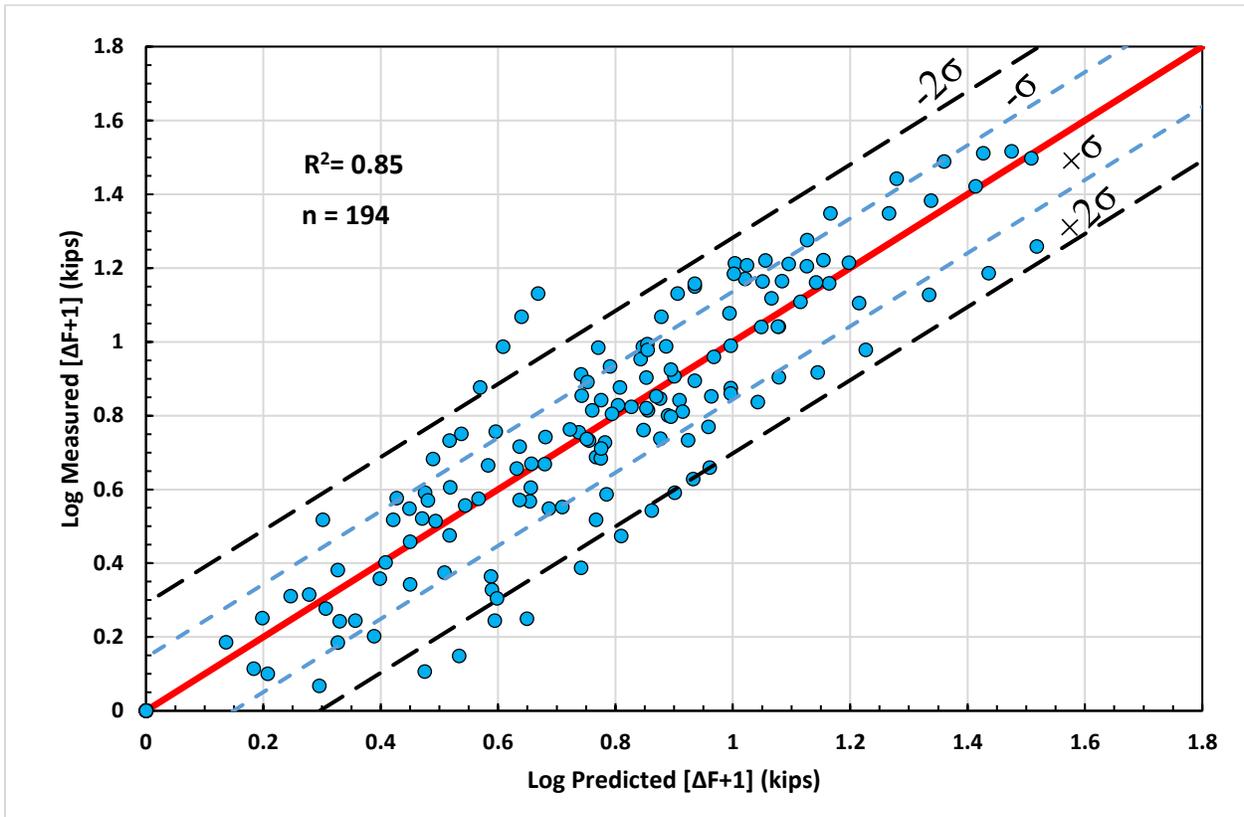


Figure 3-13: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one comparison in welded wire for laterally loaded piles with fixed head conditions.

3.2.3 Predictive equations with data for free head and fixed head conditions

Phase 3 of the test with laterally loaded piles behind the MSE wall included, 12” H piles that had fixed head conditions, and 12.75” diameter cyclically loaded piles under free head conditions. Data points from the fixed head tests were added to data points from phase 1 and 2

testing to determine if one equation could be developed to account for both pile head boundary conditions. Cyclic loading test observation points are studied separately hereafter.

After reducing the variables that had statistical significance, an R^2 value of 0.82 was obtained with eight terms in the multiple linear regression. Three of these terms had the relative compaction involved. After eliminating these terms, which had the least statistical significance, the R^2 value decreased to 0.75 but the prediction equation coefficients were the same as with Equation 3-4. This result indicates that head fixity has no effect on the maximum induced tensile force in welded wire reinforcing elements behind the MSE wall. Figure 3-14 shows a comparison of induced tensile force calculated with Equation 3-4 and Equation 2-4 for an increasing pile head load.

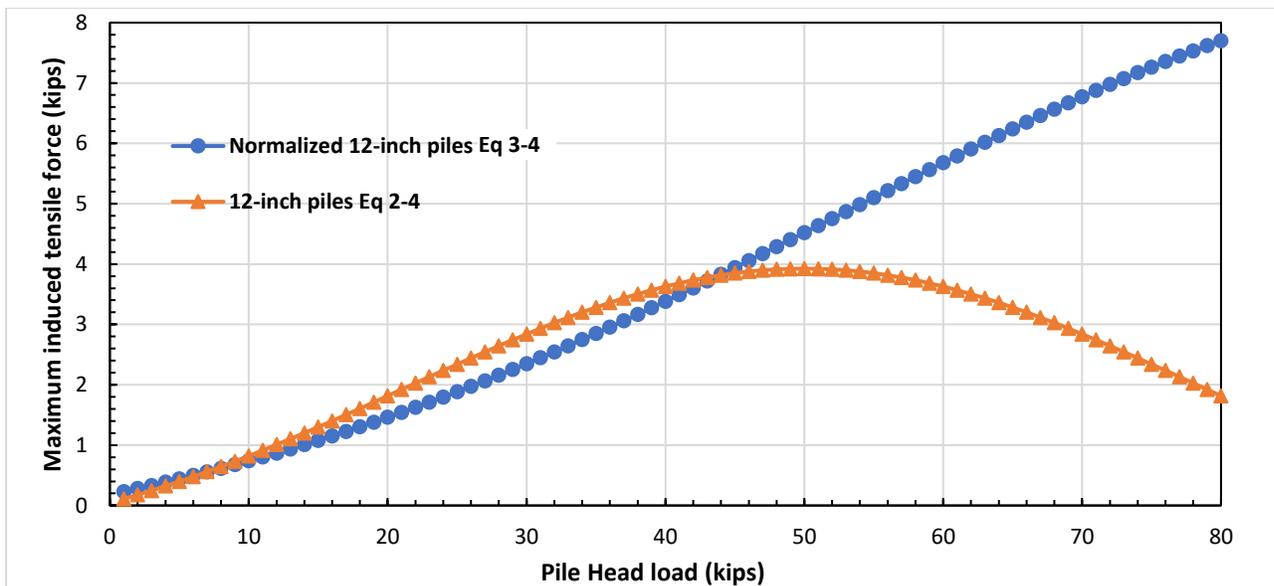


Figure 3-14: Comparison of maximum induced tensile force calculated with equation 3-4 and equation 2-4 for an increasing pile head load.

Both equations have good agreement up to a pile head load of about 40 kips; however, for higher loads Equation 2-4 inaccurately predicts a decrease in the maximum tensile force on the welded wire reinforcing grid. In contrast, Equation 3-4 provide realistic values of induced tensile force for pile head loads up to 100 kips.

Figure 3-14 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-4 including data from the laterally loaded fixed head piles behind MSE wall test. This equation, based on 1250 data points, has an R^2 value of 0.75. Blue observation points are phase 1 and phase 2 data points and red squares are observation points from the fixed head condition test. The agreement between measured and predicted values for phase 3 appear to be relatively consistent with those for phase 1 and 2 despite the higher pile head load.

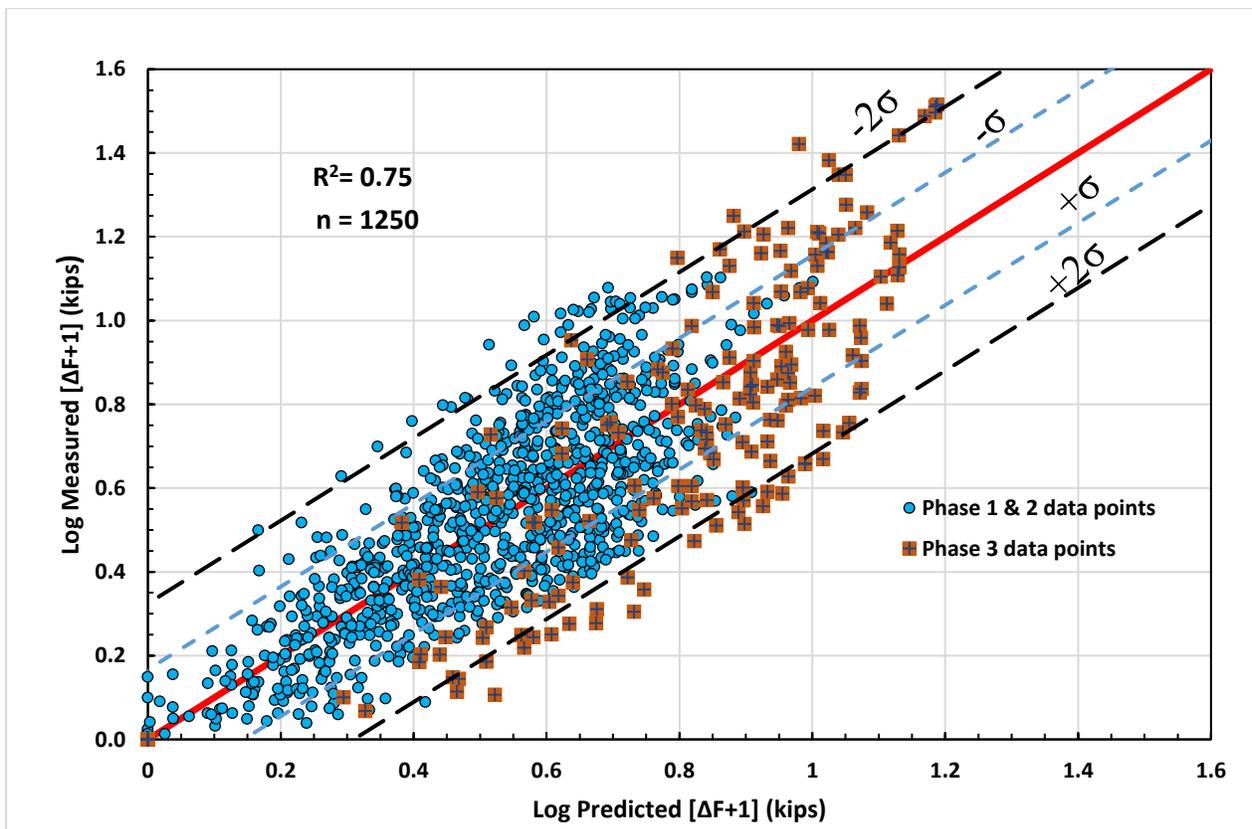


Figure 3-15: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one using data from all static loading for welded wire data.

One conclusion from the statistical analysis with the welded wire reinforcement was that relative compaction had a very high statistical significance, therefore, it seems desirable to

develop prediction equations for different levels of backfill compaction for welded wire. The Federal Highway Administration requires that the backfill in MSE walls should be compacted to the specified density, usually 95 to 100 percent of the proctor standard test maximum density and within the specified range of optimum moisture content (Federal Highway Administration, 2009). However, it can be difficult to obtain that level of relative compaction between the back face of the wall and the piles. Thus, two equations were developed, one for relative compaction less than 95 percent and another one for relative compaction of 95 percent and higher.

For backfill soil with a relative compaction less than 95%, (typically between 88 and 95%) the maximum tensile force (ΔF) induced in the reinforcements by an applied pile head load is given by the prediction equation,

$$\Delta F(kips) = 10^{[11.83 \frac{P}{P_y} - 1.93 \left(\frac{\sigma_v}{p_a}\right)^2 + 2.16 \frac{\sigma_v}{p_a} - 0.078 \frac{T}{D} - 0.059 \frac{S}{D} + 0.027]} - 1 \quad (3-6)$$

Where:

P is the pile head load in units of force

P_y is the yield force of a 12.75” diameter pile under compression in units of force (775 kips)

S is the distance from the back face of the wall to the center of the pile in units of length

D is the diameter of the pile in units of length.

σ_v is the vertical stress of the soil at the depth of the reinforcing strip in units of pressure

p_a is the atmospheric pressure (a constant value) in units of pressure.

T is the transverse distance from the center of the pile to the reinforcing grid

Figure 3-16 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-6 which is for welded wire reinforcing grid in soil with a relative compaction less than 95 percent. This equation had an R^2 of 0.78 with a standard deviation of the $\log(1+\Delta F)$ is 0.13. The

vertical stress of the soil particles at the depth of the reinforcements was the most statistically significant variable for this equation. The agreement between the measured and computed data points is good and there does not appear to be any obvious bias.

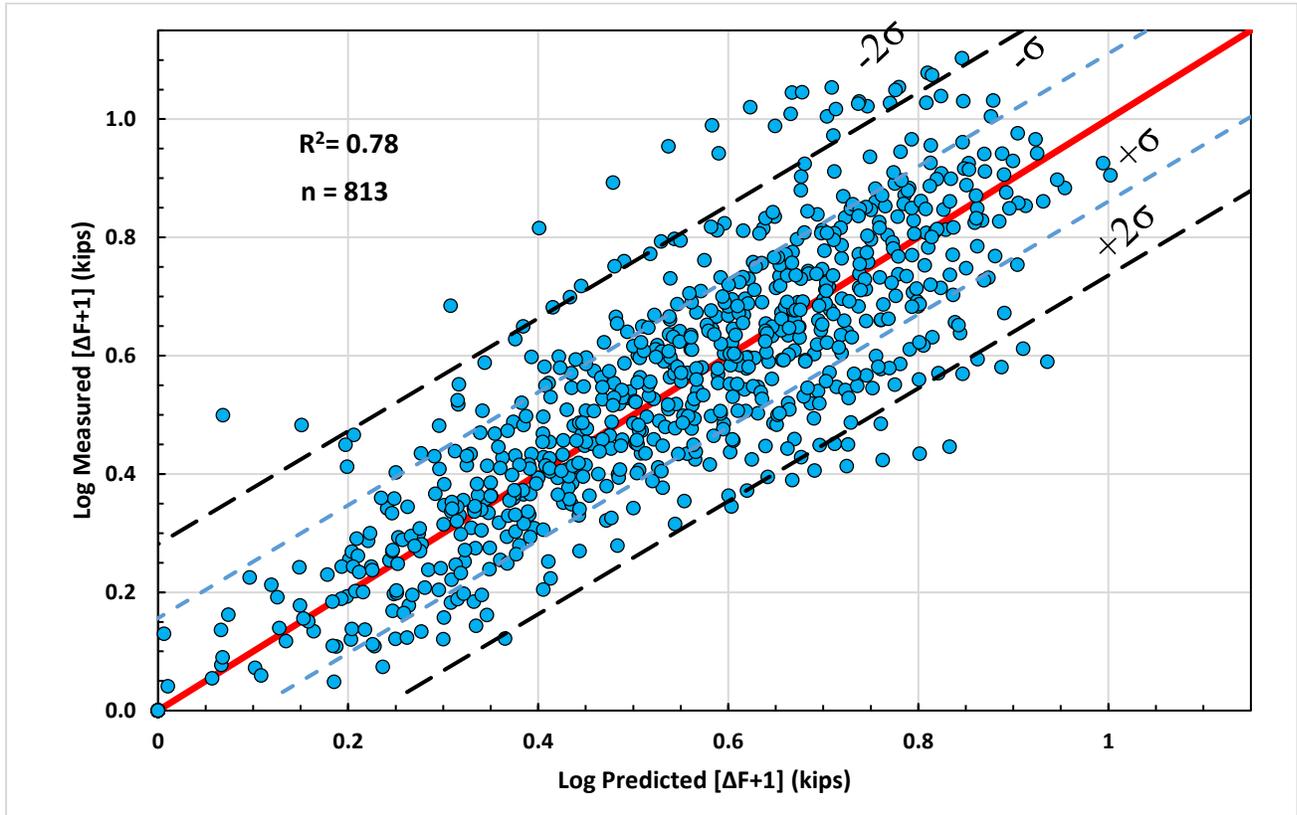


Figure 3-16: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one for welded wire with less than 95 percent relative compaction.

For backfill soil with a relative compaction greater than 95%, (typically 95 to 100%), the maximum tensile force (ΔF) induced in the reinforcements by an applied pile head load is given by the prediction equation,

$$\Delta F(kips) = 10^{[11.69 \frac{P}{P_y} - 36.82 \left(\frac{P}{P_y}\right)^2 + 1.33 \left(\frac{\sigma_v}{p_a}\right)^2 - 0.047 \frac{T}{D} - 2.28 \frac{\sigma_v}{p_a} + 0.98]} - 1 \quad (3-7)$$

Where:

P is the pile head load in units of force

P_y is the yield force of a 12.75” diameter pile under compression in units of force (775 kips)

D is the diameter of the pile in units of length consistent with T

σ_v is the vertical stress of the soil at the depth of the reinforcing strip in units of pressure

p_a is the atmospheric pressure (a constant value) in units of pressure.

T is the transverse distance from the center of the pile to the reinforcing grid in units of length

Figure 3-17 shows the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-7 which is for welded wire reinforcing grid in soil with a relative compaction of 95 percent or more. This equation has an R^2 of 0.86 with a standard deviation of the $\log(1+\Delta F)$ is 0.15. The distance between the back face of the wall and the center of the pile was not a statistically significant variable for this equation, in addition, both the vertical stress and the pile head load were the most statistically significant variables. The agreement between the measured and computed data points is good and there does not appear to be any obvious bias

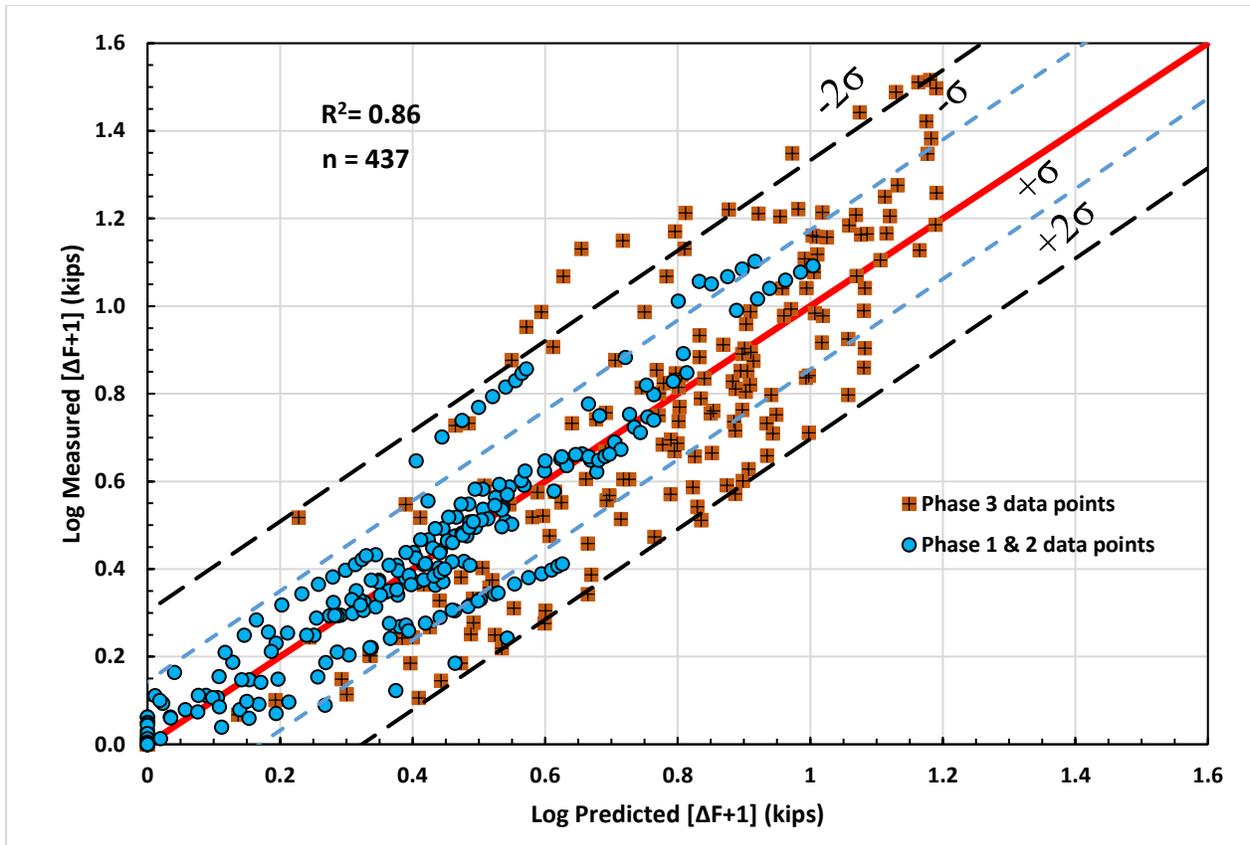


Figure 3-17: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one for welded wire with 95 percent or more relative compaction.

3.2.4 Cyclic loading test

Piles that are cyclically loaded behind an MSE wall might have a different effect on the reinforcing grid. As noted previously, Wilson (2020) reported results from full-scale tests on 12.75” diameter piles behind an MSE wall with welded wire reinforcements. Equation 3-4 was used to evaluate the need of developing a prediction equation for the cyclic loading case. Data was divided into two directions of loading because the magnitude of the induced tensile force is significantly less when loads are applied away from the wall in comparison to when loads are applied in the direction of the wall (Wilson, 2020).

Figure 3-18 illustrates the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation

3-4 with the data obtained from maximum tensile force measured for cyclically loaded piles towards the wall.

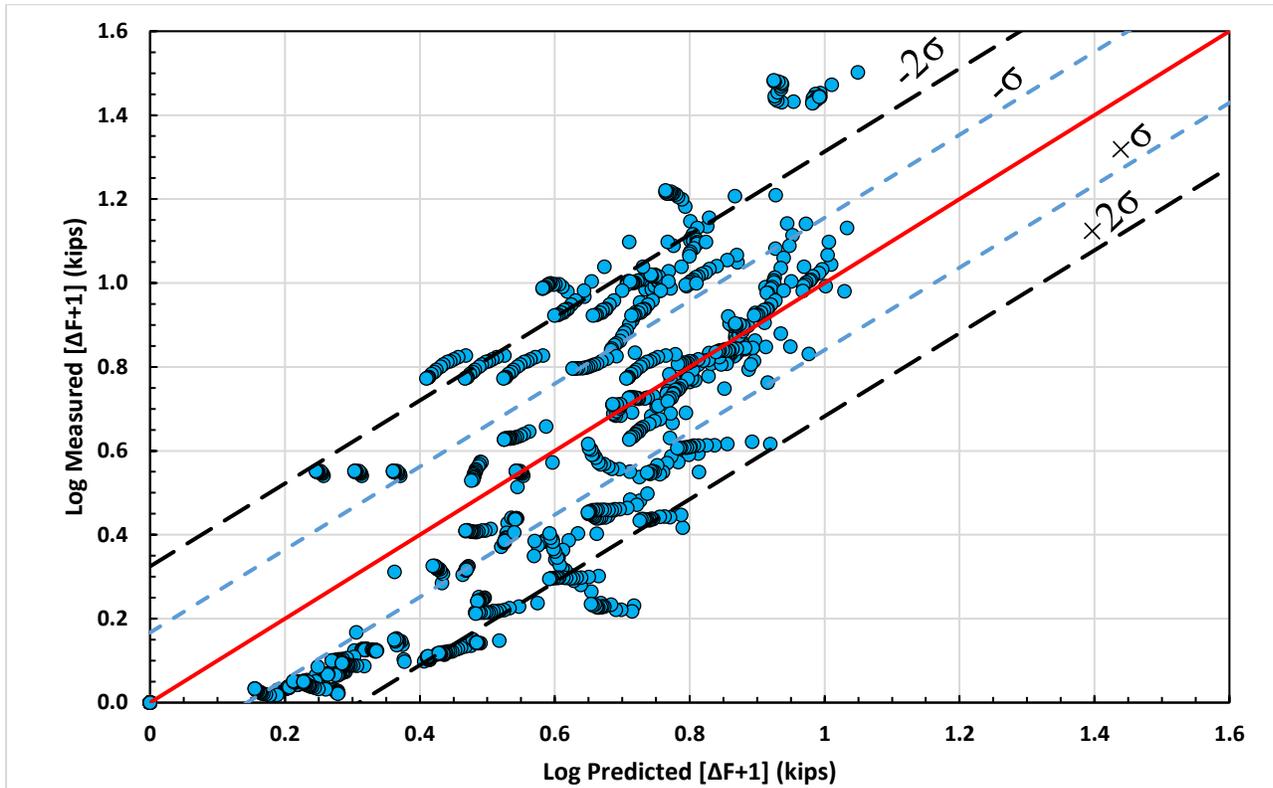


Figure 3-18: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one using Equation 3-4 with cyclically loaded piles towards the wall data.

There are 442 of the 1121 data points represented in Figure 3-18 that lie within the one standard deviation region which is 39.4% of our observation points. Additionally, there are 980 observation points within the two standard deviation region which makes 87.4% of our observation points for cyclically loaded piles towards the wall. Typically, we would expect about 68% and 95% within the one and two standard deviation boundaries, respectively if the data was in reasonable agreement with the previously developed equation. In this case, it would be necessary to expand the one standard deviation region by about 1.6 times the original standard deviation of the equation to reach 68% of the observation points in that region. Therefore, the equation for the monotonic loading condition was not particularly robust for the cyclic loading case. Therefore,

additional regression analyses were performed to improve the prediction of induced tensile force for the cyclic loading case.

A prediction equation was first developed for the cyclic loading case when the piles are loaded in the direction of the wall. This equation increases the accuracy of calculations for this case. The maximum tensile force (ΔF) induced in the reinforcements by a cyclically applied pile head load in the direction of the wall is given by the prediction equation,

$$\Delta F(kips) = 10^{[12.49 \frac{P}{P_y} - 0.171 \frac{T}{D} - 1.68 \frac{\sigma_v}{p_a} + 0.25 \left(\frac{\sigma_v}{p_a}\right) \left(\frac{T}{D}\right) + 0.98]} - 1 \dots \dots \dots (3-8)$$

Where:

P is the pile head load in units of force

P_y is the yield force of a 12.75” diameter pile under compression in units of force (775 kips)

D is the diameter of the pile in units of length consistent with T

σ_v is the vertical stress of the soil at the depth of the reinforcing strip in units of pressure

p_a is the atmospheric pressure (a constant value) in units of pressure.

T is the transverse distance from the center of the pile to the reinforcing grid in units of length

It should be noted that for this equation the distance between the back face of the wall and the center of the pile was not statically significant, thus, the equation could be reduced to four terms without significantly reducing the R^2 value. The R^2 value for equation 3-8 is 0.82 and the standard deviation of the $\log(1+\Delta F)$ is 0.17. Figure 3-19 shows the logarithm of measured maximum induced tensile force plus one in kips vs. the logarithm of the calculated maximum induced tensile force plus one using Equation 3-8 which is for welded wire reinforcing grid for laterally loaded piles cyclically loaded in the direction of the wall.

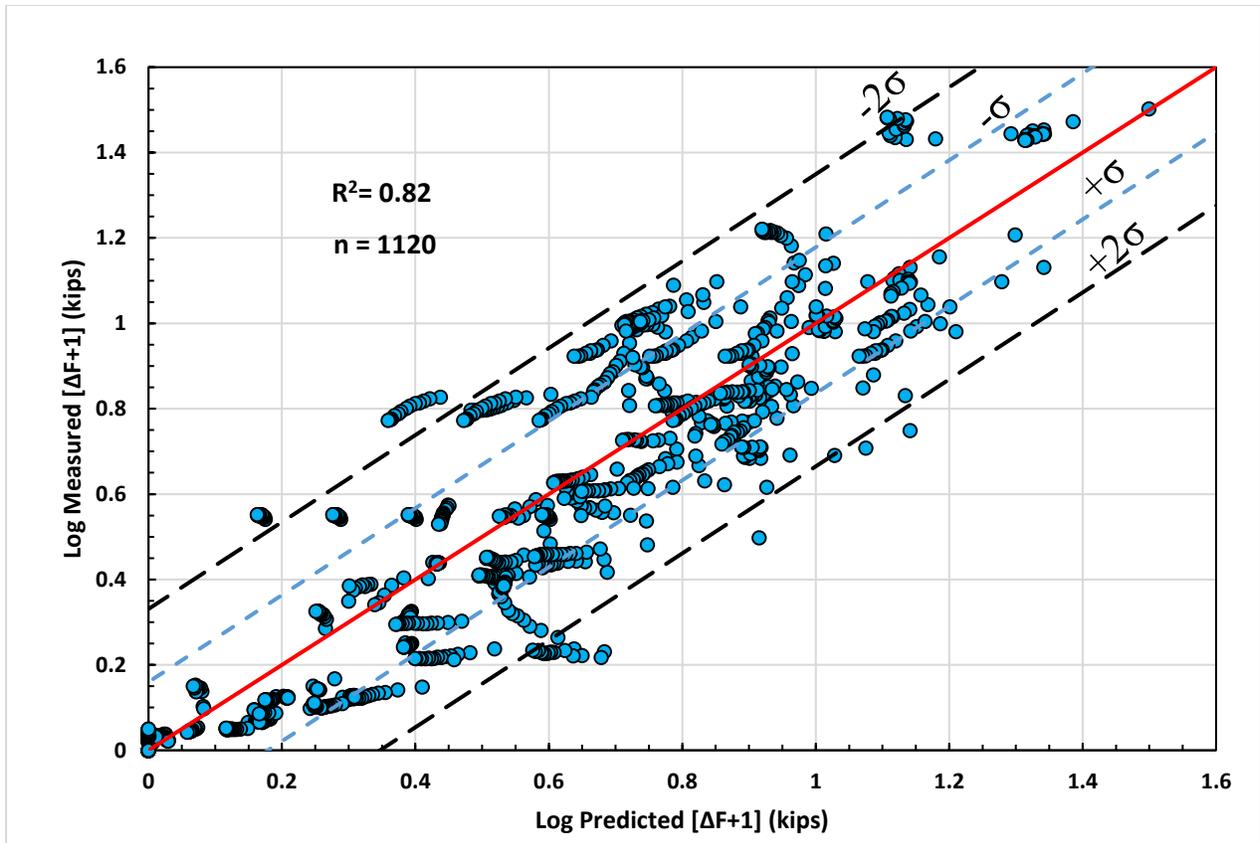


Figure 3-19: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one using equation 3-8 with cyclically loaded piles towards the wall data.

Figures 3-20 to 3-23 provide plots showing comparisons of the induced tensile force versus pile head load for the static loading (Equation 3-4) and the cyclic loading (Equation 3-8) vs pile head load. These figures show the differences in the induced force for piles located at 2, 3, 4 and 5 pile diameters, respectively from the back face of the wall to the center of the pile. The vertical stress and the transverse distance between the reinforcing grid and the center of the pile were kept constant in all cases.

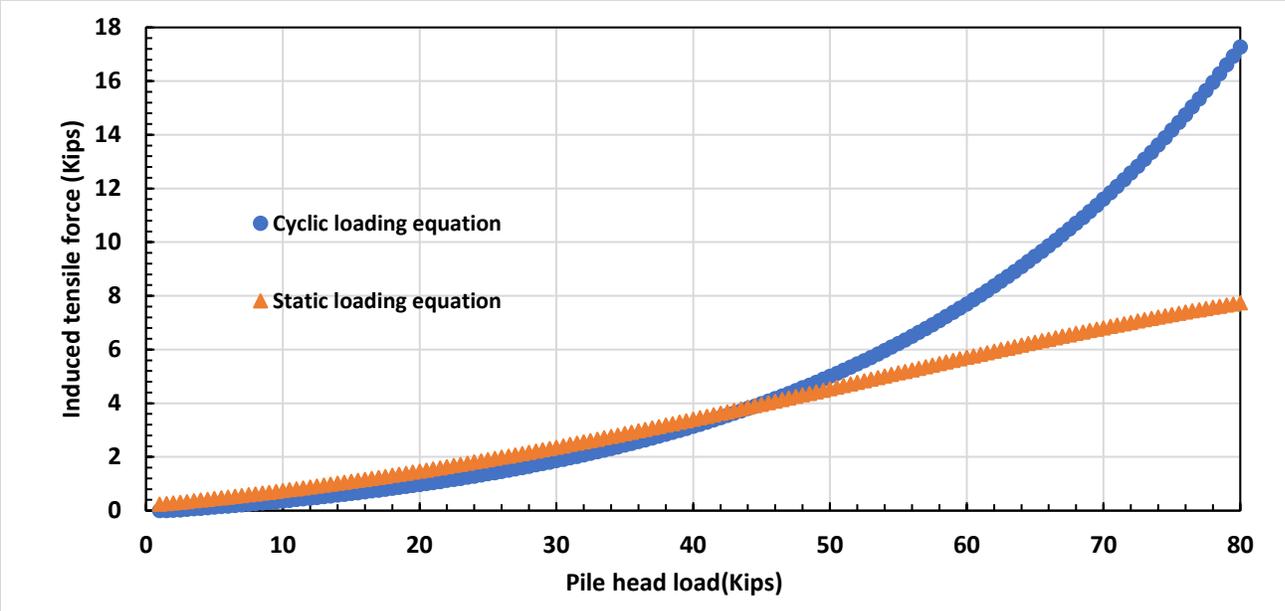


Figure 3-20: Comparison of induced tensile force with static loading equation and cyclic loading equation for a $S = 2D$

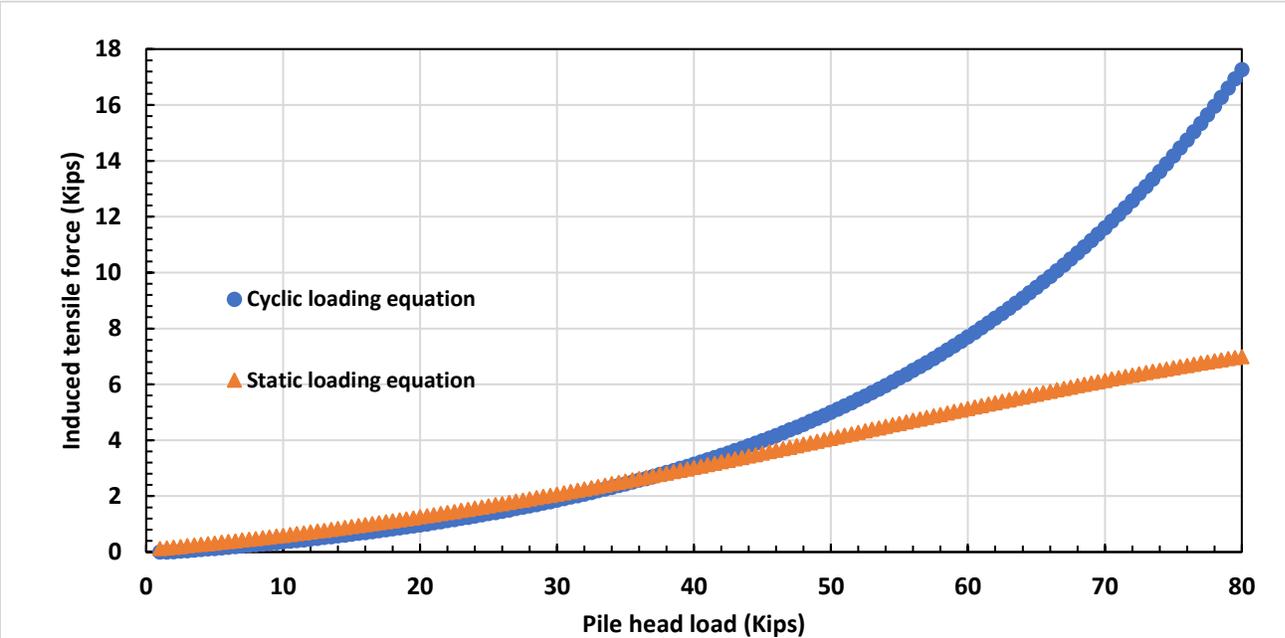


Figure 3-21: Comparison of induced tensile force with static loading equation and cyclic loading equation for a $S = 3D$

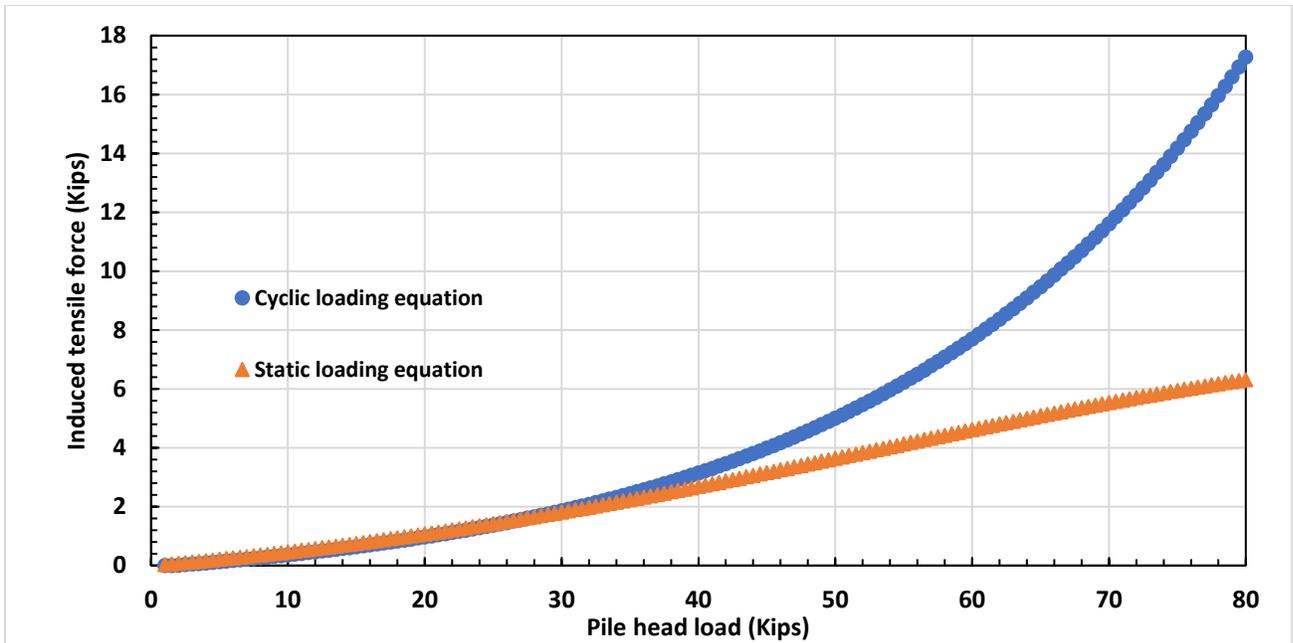


Figure 3-22: Comparison of induced tensile force with static loading equation and cyclic loading equation for a S = 4D

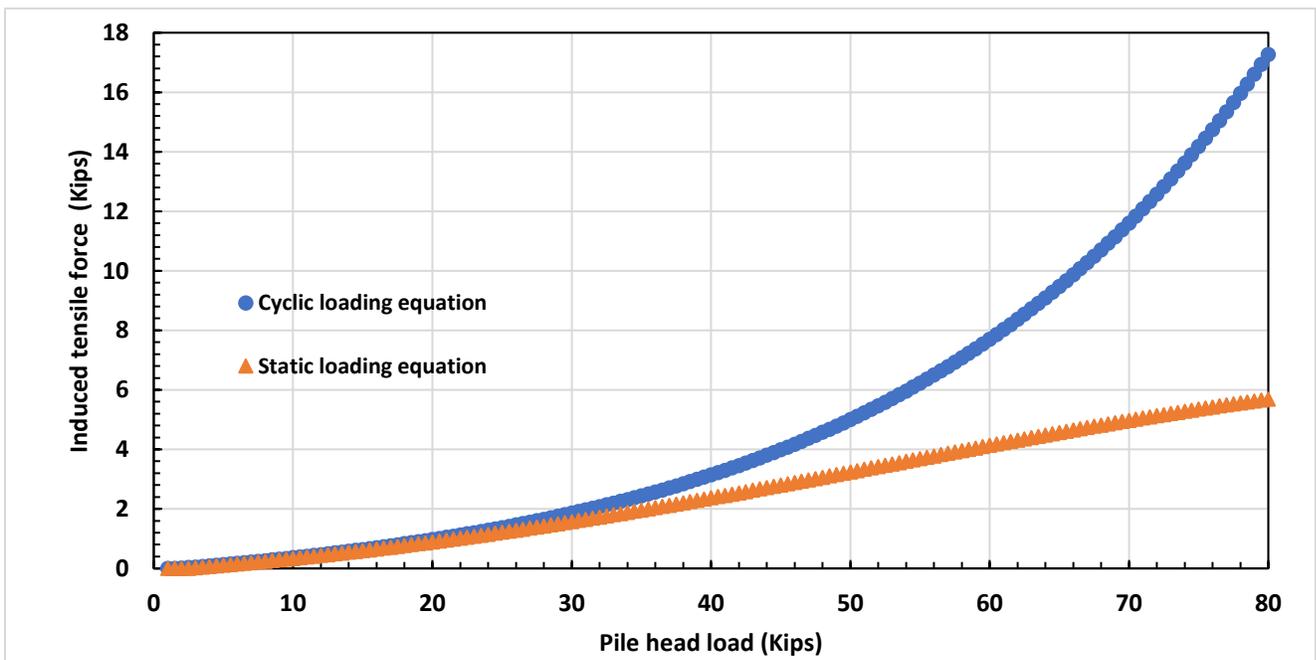


Figure 3-23: Comparison of induced tensile force with static loading equation and cyclic loading equation for a S = 5D

It can be observed that there is good agreement in calculated reinforcement force for the static and cyclic loading tests up to pile head loads of about 30 kips in all cases. As piles were

located closer to the MSE wall, good agreement was observed at progressively higher load. For example, for a pile at two pile diameters behind the wall, agreement was good up to pile head loads of about 45 kips. At higher loads, the cyclic equation produced much higher reinforcement forces than predicted by the static load equation.

Likewise, Figures 3-24 to 3-26 show comparisons between induced tensile force in the reinforcements for static and cyclic loading while keeping the distance from the back face of the wall and the center of the pile constant but increasing the vertical stress on the reinforcement. The vertical effective stress was increased from that at reinforcement depths of 1.25, 3.75 and 6.25 ft, respectively for the three figures. There is good agreement between the cyclic and static equations up to pile head loads of about 30 kips for the effective stress at a depth of 1.25 ft and the pile head load limit for good agreement increases to 40 and 48 kips at effective stresses acting on the reinforcements at 3.75 ft and 6.25 ft, respectively.

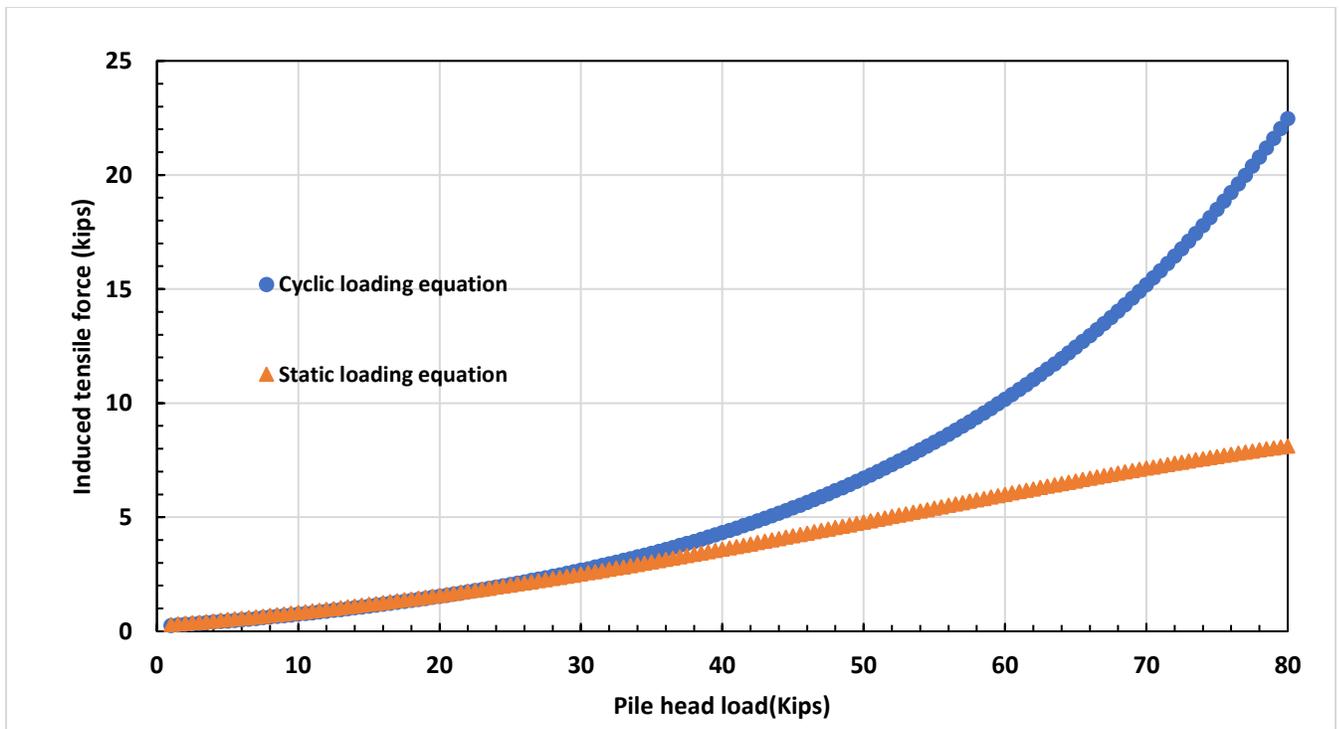


Figure 3-24: Comparison of induced tensile force with static loading equation and cyclic loading equation for a σ_v at a depth of 1.25 ft

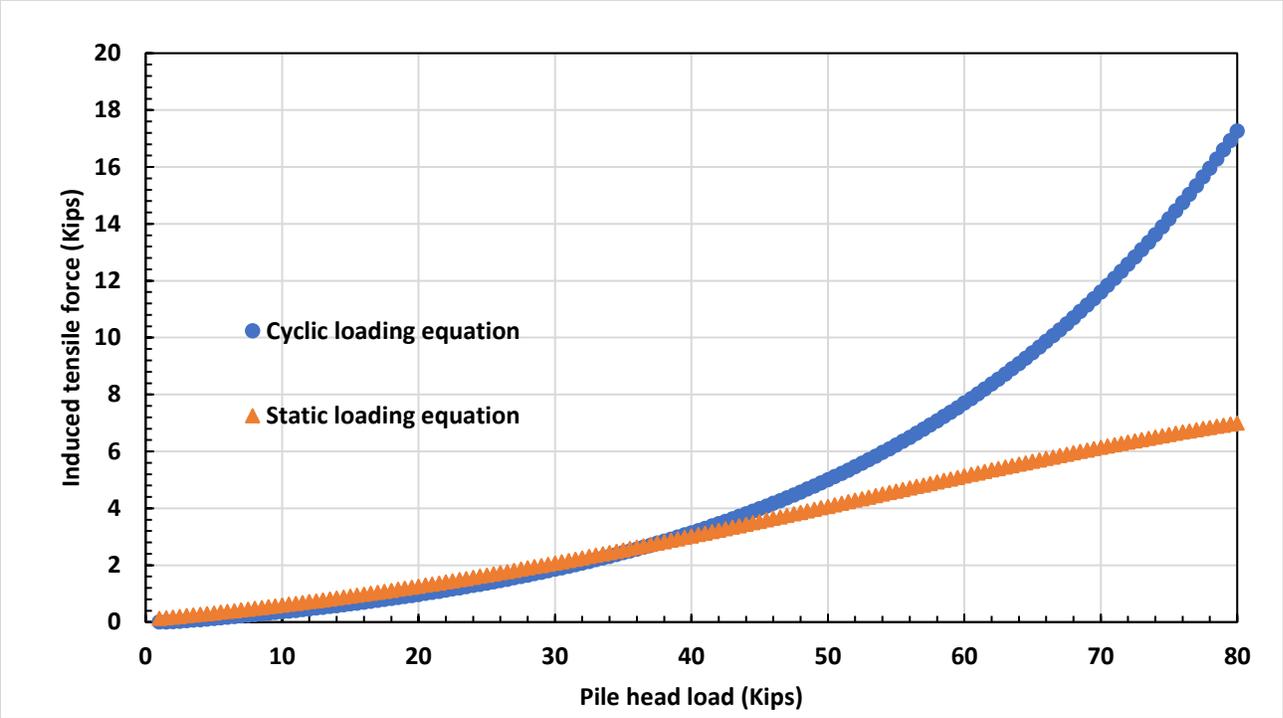


Figure 3-25: Comparison of induced tensile force with static loading equation and cyclic loading equation for a σ_v at a depth of 3.75 ft

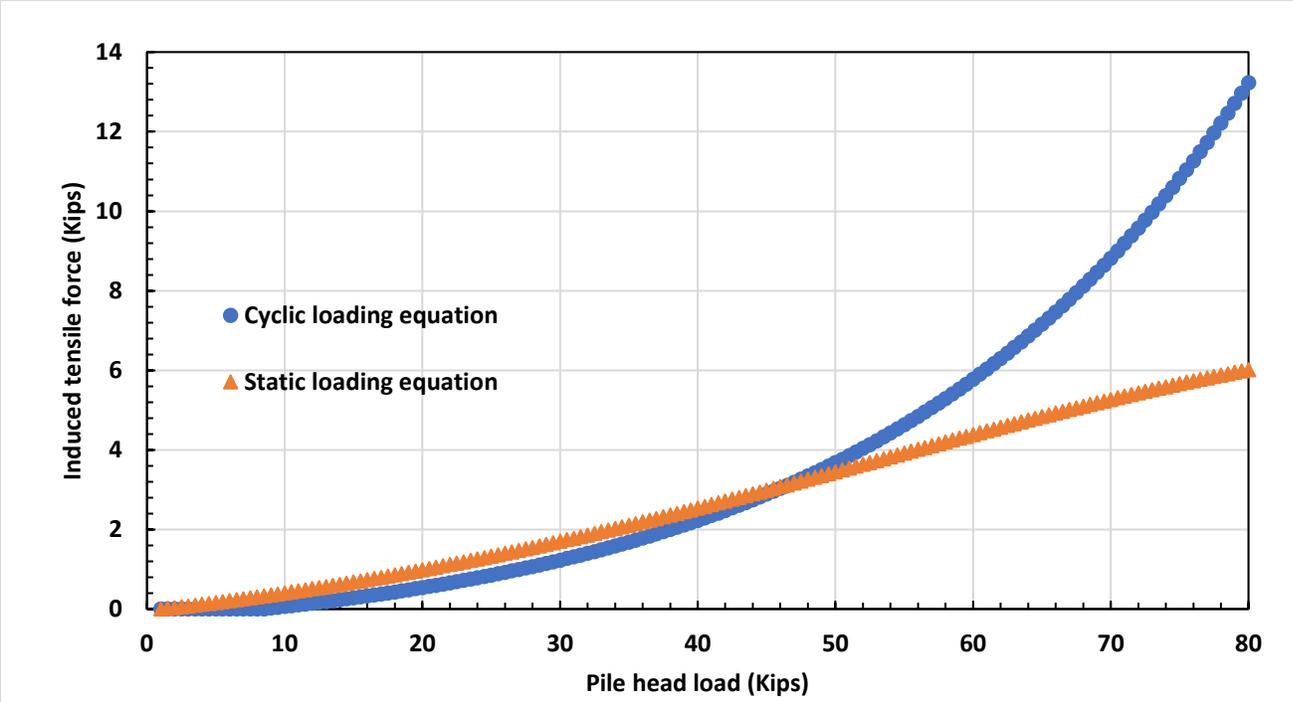


Figure 3-26: Comparison of induced tensile force with static loading equation and cyclic loading equation for a σ_v at a depth of 6.25 ft

As a final step, the induced tensile force data from the cyclic loading tests were added to the tensile force data for the static loading tests involving free head and fixed head piles. Multi-variable regression analyses were performed and the maximum tensile force (ΔF) induced in the welded-wire reinforcements by an applied pile head load was given by the prediction equation,-

$$\Delta F(kips) = 10^{[15.89 \frac{P}{P_y} - 56.28 \left(\frac{P}{P_y}\right)^2 - 0.051 \frac{T}{D} - 0.556 \frac{\sigma_v}{p_a} - 0.021 R_c + 2.4]} - 1 \quad (3-9)$$

Where:

P is the pile head load in units of force

P_y is the yield force of a 12.75” diameter pile under compression in units of force (775 kips)

D is the diameter of the pile in units of length, consistent with T

σ_v is the vertical stress of the soil at the depth of the reinforcing strip in units of pressure

p_a is the atmospheric pressure (a constant value) in units of pressure.

T is the transverse distance from the center of the pile to the reinforcing grid

R_c is the relative compaction of the soil between the back face of the pile and the pile

The R^2 value is 0.75 and the standard deviation is 0.18 for a data set consisting of 2370 observations. It is noted that the relative compaction was more statistically significant than the distance between the back face of the wall and the center of the pile. This is due to the importance of the level of compaction in the test. For the cyclic loading test, the soil between the wall and the pile had a relative compaction of 95 percent in comparison to the maximum density for the standard Proctor test.

Figure 3-27 compares the logarithm of measured maximum induced tensile force plus one in kips vs. the logarithm of the calculated maximum induced tensile force plus one using equation 3-9 with all the data from laterally loaded piles behind the MSE wall with welded wire reinforcing grid.

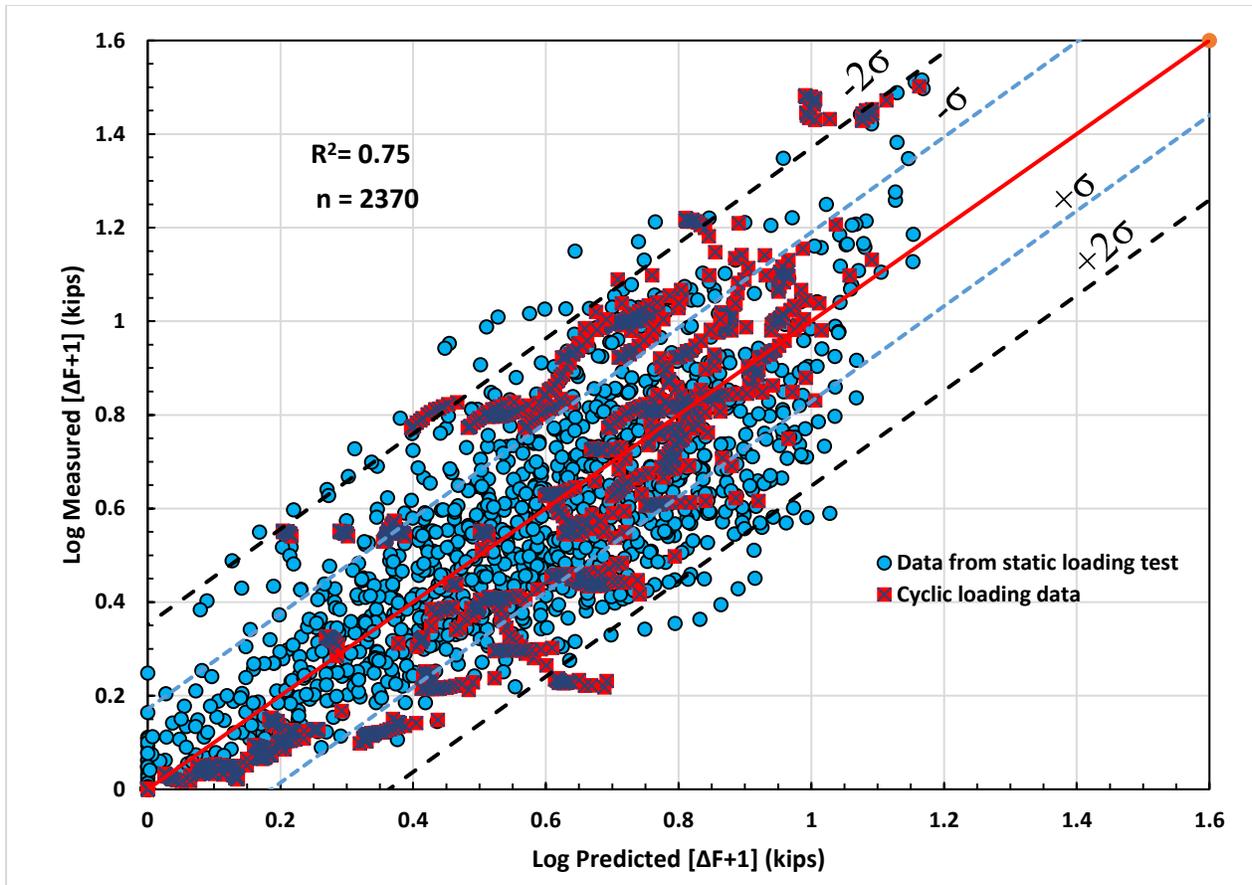


Figure 3-27: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one using equation 3-9 with all data from tests involving welded wire.

Data from the cyclic loading test did not decrease the R^2 from the equation but it did increase the standard deviation as more data was added. Nevertheless, Equation 3-9 provides a reasonable estimation of the measured force considering that the data includes test results from static and cyclic loadings as well as free head and fixed head boundary conditions.

Figures 3-27 to 3-29 provide a comparison of the difference in the computed induced tensile force with increasing pile head load for different vertical stress on the reinforcement, which was a parameter with the most influence on the results.

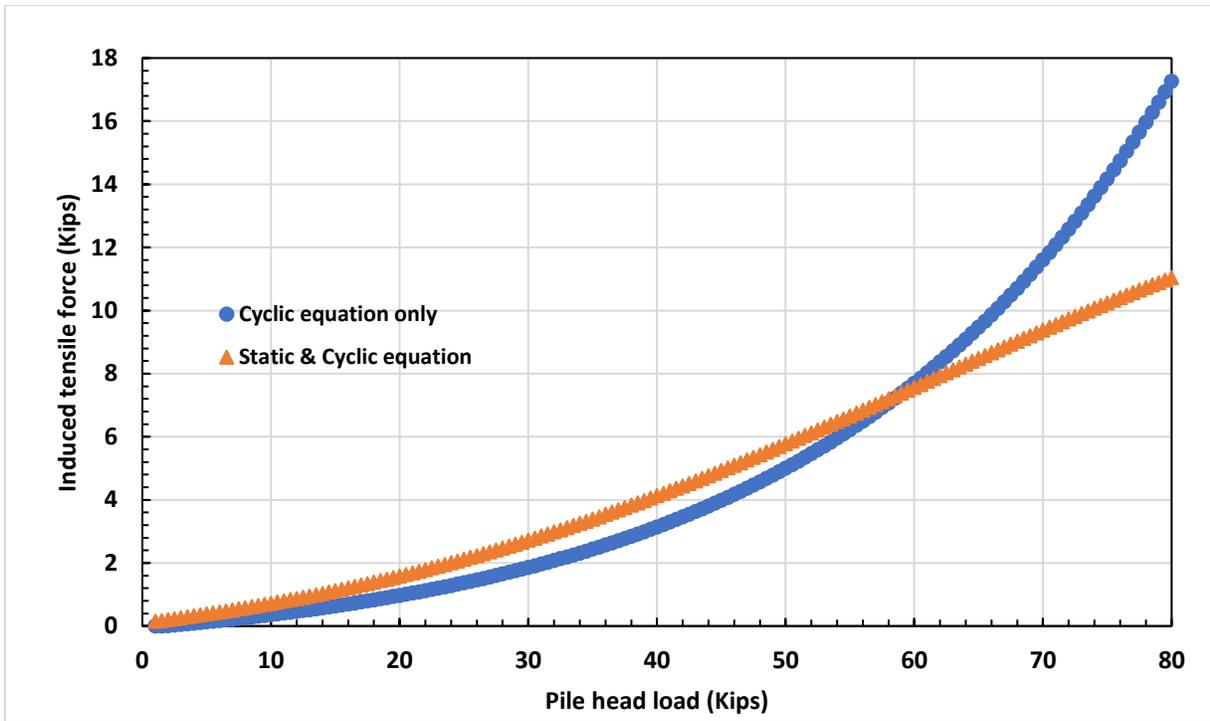


Figure 3-28: Comparison of induced tensile force with static & cyclic loading equation and only cyclic loading data equation for σ_v at a depth of 1.25 ft.

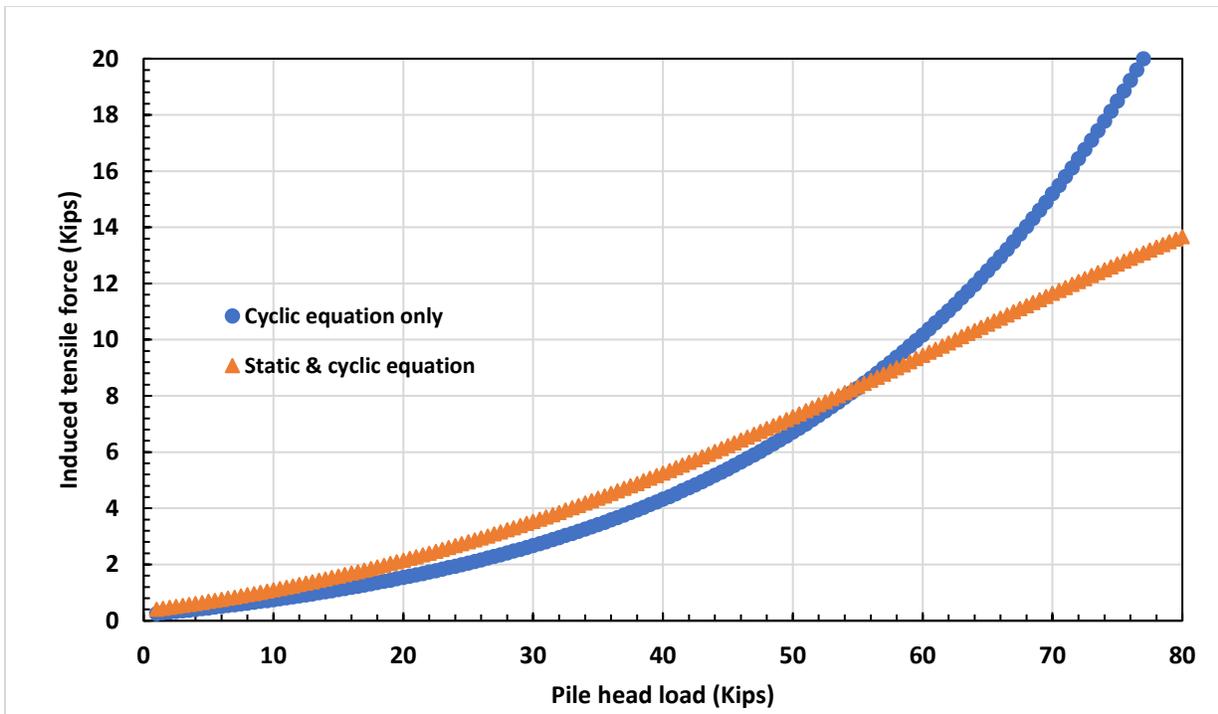


Figure 3-29: Comparison of induced tensile force with static & cyclic loading equation and only cyclic loading data equation for σ_v at a depth of 3.75 ft.

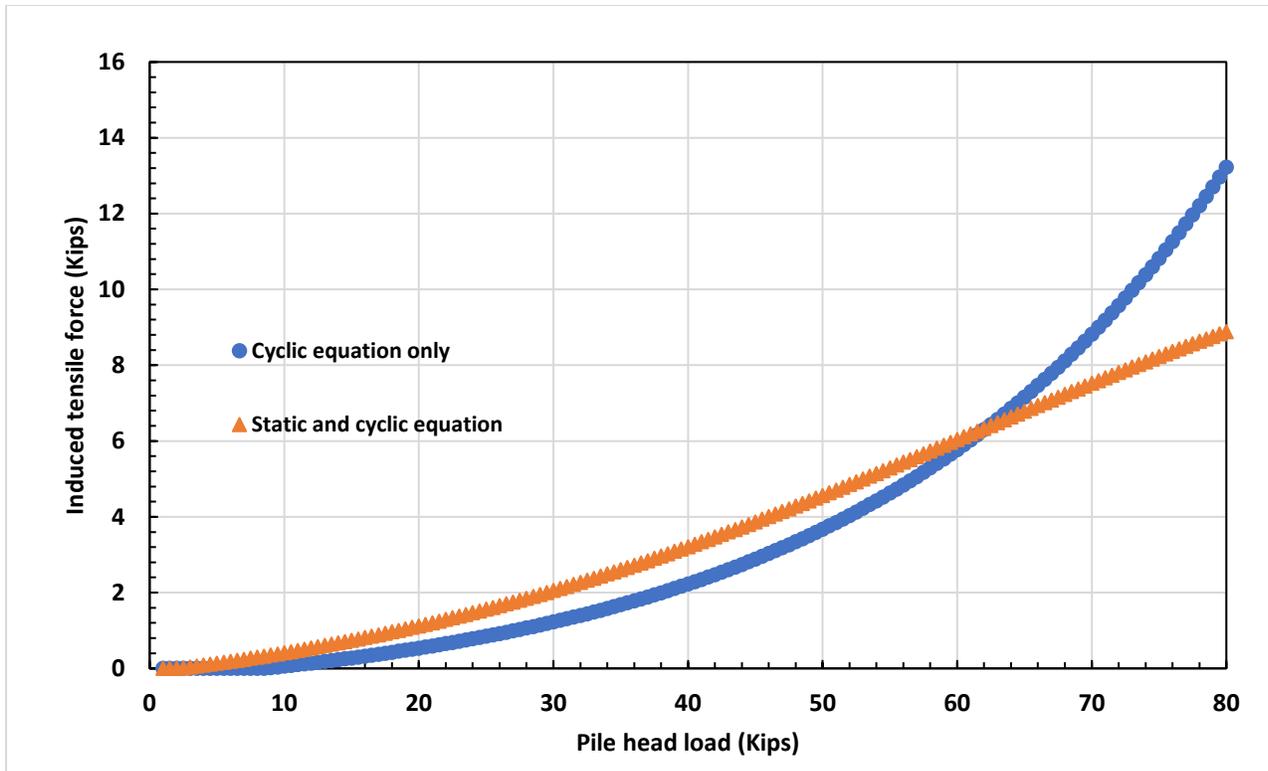


Figure 3-30: Comparison of induced tensile force with static & cyclic loading equation and only cyclic loading data equation for a σ_v at a depth of 6.25 ft

It can be observed that adding cyclic loading data into the prediction equation with static loading leads to reasonable estimation of the induced force up pile head loads of about 60 kips compared with the equation with only cyclic loading data. Above this load the cyclic load exceeds the predicted value using the equation for the entire data set. For pile head loads less than 60 kips, the disagreement appears to increase as the vertical stress increases to that a depth of 6.25 ft.

Another condition that was evaluated separately is the cyclic loading of piles behind the MSE wall in the direction away from the wall. Data obtained from the full-scale load test was compared with the induced force predicted by Equation 3-4 to determine if this equation would provide reasonable agreement.

Figure 3-31 illustrates the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-4 with data from cyclic laterally loaded piles in the direction away from the wall.

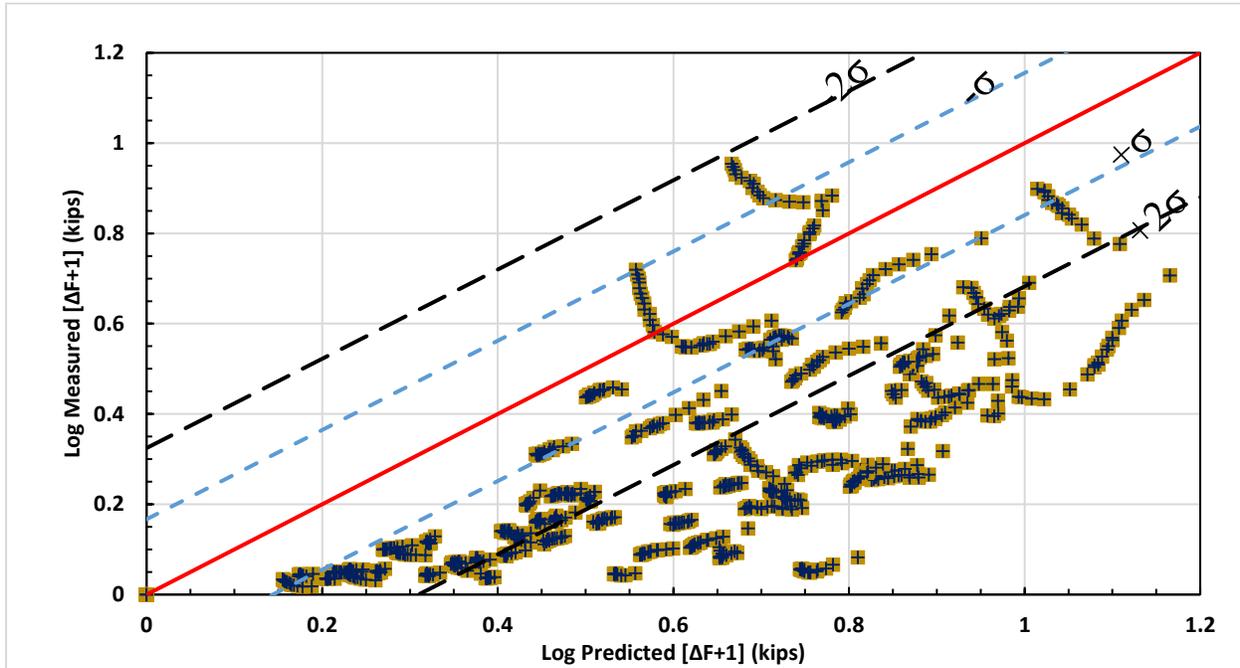


Figure 3-31: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one using equation 3-4 with cyclic loading away from the wall test data.

It can be observed that most of the data points from this comparison are below the plus one to two standard deviation line. This fact leads to the conclusion that Equation 3-4 overpredicts the measured induced force for piles loaded cyclically away from the wall. This is due to the increased volume of soil that is available to carry the lateral stresses in contrast to the case when loading towards the wall. As a result, the reinforcements are not strained as much, thus, this is not likely to be a critical condition.

A prediction equation was also developed from data for cyclic loading away from the wall. In this case, the maximum tensile force (ΔF) induced in the reinforcements by an applied pile head load is given by the prediction equation,

$$\Delta F(kips) = 10^{[4.88 \frac{P}{P_y} + 0.081 \left(\frac{S}{D}\right)^2 - 0.51 \frac{S}{D} - 0.037 \frac{T}{D} - 0.7 \frac{\sigma_v}{p_a} + 1.25]} - 1 \quad (3-10)$$

Where:

P is the pile head load in units of force

P_y is the yield force of a 12.75” diameter pile under compression in units of force (775 kips)

S is the distance from the back face of the wall to the center of the pile in units of length

D is the diameter of the pile in units of length.

σ_v is the vertical stress of the soil at the depth of the reinforcing strip in units of pressure

p_a is the atmospheric pressure (a constant value) in units of pressure.

T is the transverse distance from the center of the pile to the reinforcing grid

This equation has an R² value of 0.69 and a standard deviation of the log(1+ΔF) is 0.1. This shows that the data correlates fairly well and the prediction equation has a lower standard deviation than other developed equations. In addition, it is noted that for this condition the spacing between the back face of the wall and the center of the pile has a high statistical significance.

Figure 3-32 illustrates the logarithm of measured maximum induced tensile force plus one in kips vs the logarithm of the calculated maximum induced tensile force plus one using Equation 3-4 with data from cyclic laterally loaded piles in the direction away from the wall. The maximum calculated value of induced tensile force in reinforcing welded wire grid for this condition was of about 5.3 kips.

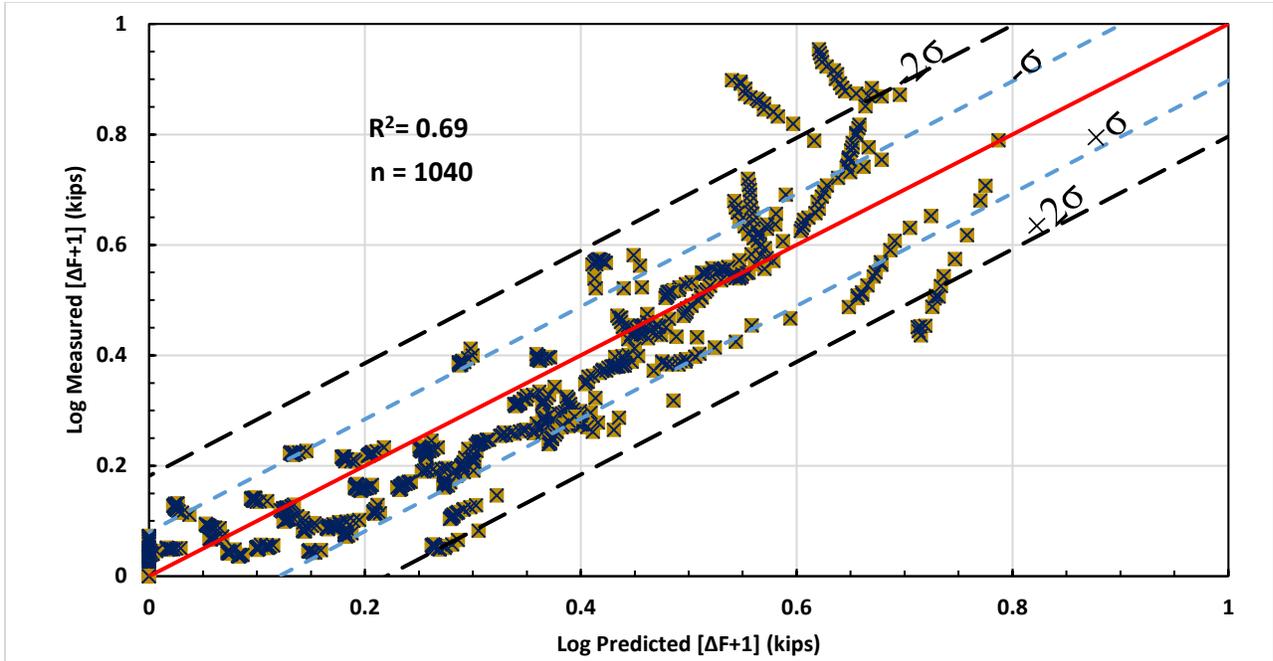


Figure 3-32: Logarithm of measured induced tensile force plus one vs logarithm of predicted induced tensile force plus one for cyclic loading away from the wall.

4 SUMMARY AND CONCLUSIONS

4.1 Summary

In this study we collected data defining induced tensile force in reinforcing elements behind MSE walls produced by laterally loaded piles. We then performed multivariable regression analyses to develop equations for predicting the maximum tensile force based on statistically significant parameters. Separate equations were developed for ribbed-strip reinforcements and welded-wire reinforcements. Previous equations were developed with full-scale tests primarily involving piles with diameters/widths of about 12 inches and without considering relative compaction, fixed head conditions, or cyclic loading effects. The newly developed empirical equations can be used to obtain estimates of the maximum induced tensile force, including variations in relative compaction, pile head boundary condition, pile diameter and type of loading. Regression equations are provided for each class of conditions, but general equations which consider all the available loading conditions were successfully developed for both ribbed-strip (Equation 3-4) and welded-wire (Equation 3-9) reinforcements.

4.2 Conclusions

1. For the prediction of induced tensile forces, the diameter of the pile is a statistically significant variable. Thus, for larger diameters the developed equation predicts smaller induced tensile force than the previous equation. The equation for induced force with ribbed strip reinforcement includes diameter effects for piles up to 24 inches in diameter while this effect is not considered for the welded-wire reinforcements owing to the lack of test data.

2. Fixed head conditions did not influence the induced tensile force on reinforcing soil elements relative to free head pile loading. Therefore, general equations can be developed which consider data from both fixed and free head conditions.
3. The relative compaction of the backfill around MSE reinforcements has a strong impact on the measured induced tensile force for welded wire reinforcements. Equations were developed for relative compaction of less than 95 percent and greater than 95 percent.
4. Cyclic loading does not have a significant impact on the prediction of induced tensile force; however, with this data, the relative compaction becomes more statistically significant. Therefore, it was possible to produce one general equation that accounted for both cyclic and static loading as well as free and fixed head loading conditions with the welded-wire reinforcements.
5. A better combination of explanatory variables was found for the prediction equation for phase 1 and phase 2 tests involving welded-wire reinforcements. This led to an equation with an R^2 of 0.74, a standard deviation of 0.38 kips, and an increased range of applicability for pile head load.

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