

## Transportation Pooled Fund Program

<b>Project Title: HY-8 Culvert Analysis Program – Phase Three of Development Efforts</b>		
<b>Project Manager and Phone Number:</b>  Mr. Joe Krolak Senior Hydraulics Engineer FHWA Office of Bridge Technology, E75-328 1200 New Jersey Avenue, S.E. Washington DC 20590 202/366/4611 <a href="mailto:joe.krolak@dot.gov">joe.krolak@dot.gov</a>	<b>Project No:</b>  TPF 5 (202)	Project is:  <input type="checkbox"/> PLANNING <input checked="" type="checkbox"/> R&D
<b>Reporting Period:</b>  FY 2010, second quarter (Jan-Mar)	<b>Multi Year Project</b>  Yes	
<b>Description of Work Performed and Progress:</b> <ul style="list-style-type: none"> <li>Completed interim maintenance effort. FHWA will post update on Hydraulic website in April 2010.</li> <li>Received report describing theoretical approach for performing hydraulic jumps (attached).</li> <li>All acquisition documents prepared. Procurement for main tasks going through FHWA acquisition process. RFP to be released under NHI IDIQ contract vehicle.</li> </ul>		
<b>STATUS AND COMPLETION DATE</b> <p style="text-align: center;">Percentage of work completed to date for total project Project is: 5%</p> <p style="text-align: center;">_____ on schedule <input checked="" type="checkbox"/> behind schedule, explain:</p> <ul style="list-style-type: none"> <li>Acquisition process taking longer than expected. Anticipate contract award early in next quarter (April 2010).</li> </ul> <p style="text-align: center;">Expected Completion Date: Summer 2012</p>		

Joe Krolak, 25 March 2010

## Hydraulic Jumps and HY-8

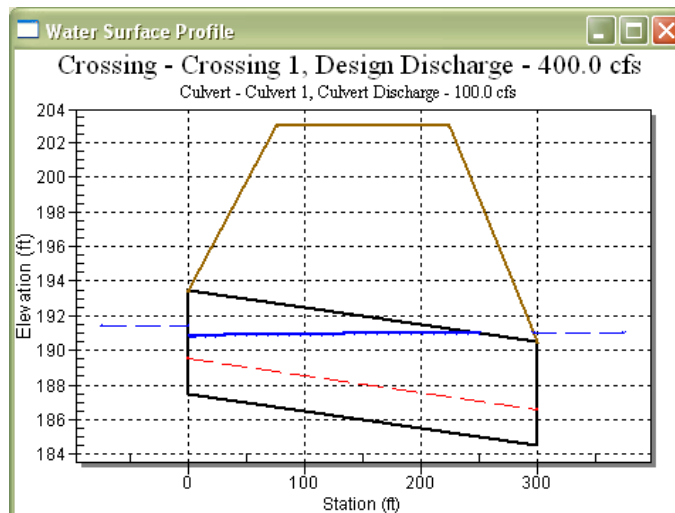
The purposes of this brief report are to (1) describe how HY-8 v 7.2 compensates for not calculating whether or not a hydraulic jump occurs in a culvert barrels, and (2) summarize completed work that will contribute towards implementing hydraulic jump analysis in straight culverts.

### Compensating for non-Hydraulic Jump Prediction

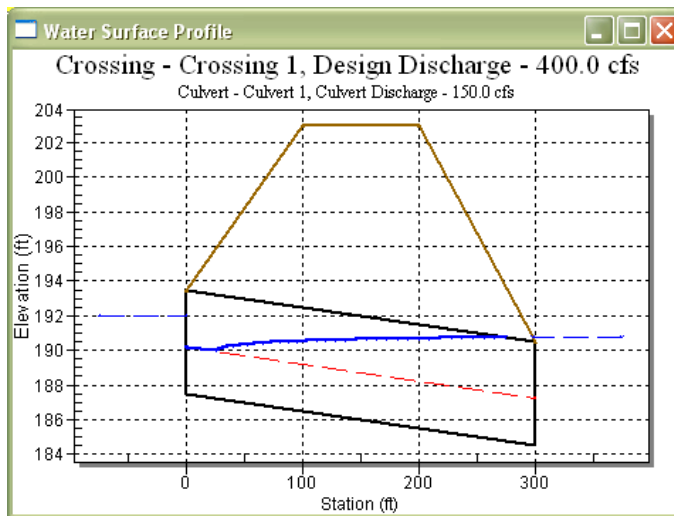
HY-8 v 7.2 does not predict whether or not a hydraulic jump will occur in a culvert barrel. It instead compensates as described below. As noted, this can result in inaccurate predictions of outlet velocity that may impact streambed stability downstream from the culvert outlet. We believe that hydraulic jumps should be implemented in HY-8.

When the tailwater elevation exceeds critical depth in the culvert, one of the following cases is assumed in HY-8 version 7.1:

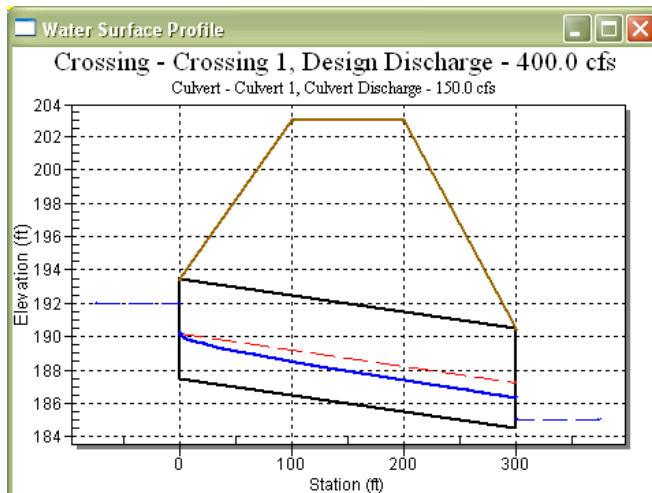
1. If the tailwater elevation is higher than the top of the culvert at the outlet ( $TW > D$ ), an S1 profile is computed from downstream to upstream. The outlet velocity is computed using full barrel area.
  - a. If the S1 profile intersects the culvert face, this outlet control HW is compared with the inlet control HW to determine which governs. The outlet control HW assumption is reasonable.



- b. If the S1 profile intersects the critical depth within the culvert barrel, the profile continues at critical depth until the entrance. Inlet control is assumed. While the HW elevation is correct, the outlet velocity is only correct if the hydraulic jump is contained in the barrel. Momentum should be checked to determine if the jump is pushed out of the culvert. This is a complex case that would require the computation of both an S2 and S1 curve to determine where the jump occurs. A benefit of this calculation would be a more realistic plot of the profile.



2. If the tailwater elevation is lower than the top of the culvert at the outlet ( $D > TW$ ), an S2 drawdown curve is plotted within the barrel. Inlet control HW is calculated. Outlet velocity is calculating using the last depth in the S2 profile.
  - a. If tailwater is lower than critical depth, the outlet velocity is reasonable. This case is shown below.



- b. If the tailwater is higher than critical depth, HY-8 assumes that the jump does not occur in the culvert and this case is treated the same as case 2(a). This conservative assumption is used in version 6.2 and 7 for computing outlet velocity. If TW does force the jump to occur in the culvert, the outlet velocity can be significantly lower. Previous versions of HY-8 computed an S1 profile. Momentum should be checked to determine if the tailwater is pushed out of the culvert. This is another complex case that would require the computation of both an S2 and S1 curve to determine where the jump occurs.

The outlet velocity can be substantially different for the cases above. The outlet velocity for the S1 curve ( $TW > D$ ) is 9.28 ft/s, while in the S2 case it is 13.54 ft/s. For case 1(b) if the jump is pushed out of the culvert, we are underreporting the outlet velocity. For case 2(b) if the jump is pushed into the culvert, we are over reporting the outlet velocity.

To remedy this situation we must consider the occurrence of a hydraulic jump. To do so, we need expressions that relate sequent depths across a hydraulic jump for both complete and incomplete jumps. A complete jump is one whose subcritical depth is less than the barrel rise; an incomplete jump is when the subcritical depth exceeds the barrel rise.

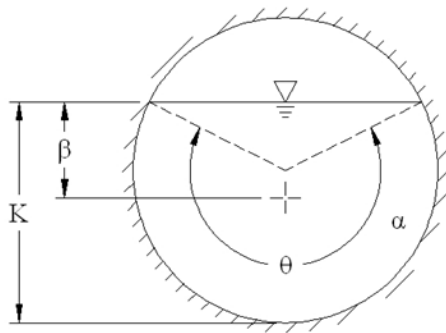
Explicit sequent depth expressions only exist (to our knowledge) for complete and incomplete jumps in a rectangular barrel. We will now summarize work done to predict sequent depths for any current culvert shape in HY-8 – for both complete and incomplete hydraulic jumps.

### Sequent Depths in HY-8 Culverts

According to HEC-14, the subcritical sequent depth ( $y_2$ ) of any supercritical depth ( $y_1$ ) may be derived from the momentum equation. However, the derivation will vary depending on the shape of the culvert as well as whether or not a complete jump is possible within the culvert barrel. The following table lists the dimensionless formulas, derived from the momentum equation, which may be used to calculate the subcritical sequent depth for any of the culvert shapes currently used in HY-8. The variables  $\alpha$  and  $\beta$ , defined as the dimensionless area and centroid depth, respectively, will subsequently be defined for each culvert shape.

Table 1. General sequent depth ratio formulas

Complete Jumps	Transition	Incomplete Jumps
$\Lambda = f(Z^2, K_1)$ $Z^2 = \frac{\alpha_1 \alpha_2 (\alpha_1 \beta_1 - \alpha_2 \beta_2)}{\alpha_1 - \alpha_2}$	$\Lambda = f(Z^2, K_1)$ $Z^2 = \frac{\alpha_1 \alpha_2 (\alpha_1 \beta_1 - \alpha_2 \beta_2)}{\alpha_1 - \alpha_2}$	$\Lambda = \frac{1}{K_1} \left\{ \begin{array}{l} 1 - \beta^{K=1} \\ + \frac{1}{\alpha_2} \left[ \begin{array}{l} \alpha_1 \beta_1 \\ - \frac{Z^2 (\alpha_1 - \alpha_2)}{\alpha_1 \alpha_2} \end{array} \right] \end{array} \right\}$
$\alpha_1 = \alpha  ^{K=K_1}$	$\alpha_1 = \alpha  ^{K=\frac{1}{\Lambda}}$	$\alpha_1 = \alpha  ^{K=K_1}$
$\alpha_2 = \alpha  ^{K=K_1 \Lambda}$	$\alpha_2 = \alpha  ^{K=1}$	$\alpha_2 = \alpha  ^{K=1}$
$\beta_1 = \beta  ^{K=K_1}$	$\beta_1 = \beta  ^{K=\frac{1}{\Lambda}}$	$\beta_1 = \beta  ^{K=K_1}$
$\beta_2 = \beta  ^{K=K_1 \Lambda}$	$\beta_2 = \beta  ^{K=1}$	
$Z^2 \equiv \frac{Q^2}{gB^2D^3}$		
$\alpha \equiv \frac{A}{BD} = f(\text{shape}, K_1, \Lambda)$		
$\beta \equiv \frac{h_c}{D} = f(\text{shape}, K_1, \Lambda)$		
$\Lambda \equiv \frac{y_2}{y_1}$		
$K_1 \equiv \frac{y_1}{D}$		



$$\alpha = \frac{1}{8}(\theta - \sin\theta)$$

$$\beta = \frac{1}{24\alpha} \left[ 3\sin\left(\frac{\theta}{2}\right) - \sin^3\left(\frac{\theta}{2}\right) - 3\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) \right]$$

$$\theta = 2\cos^{-1}[1 - 2K]$$

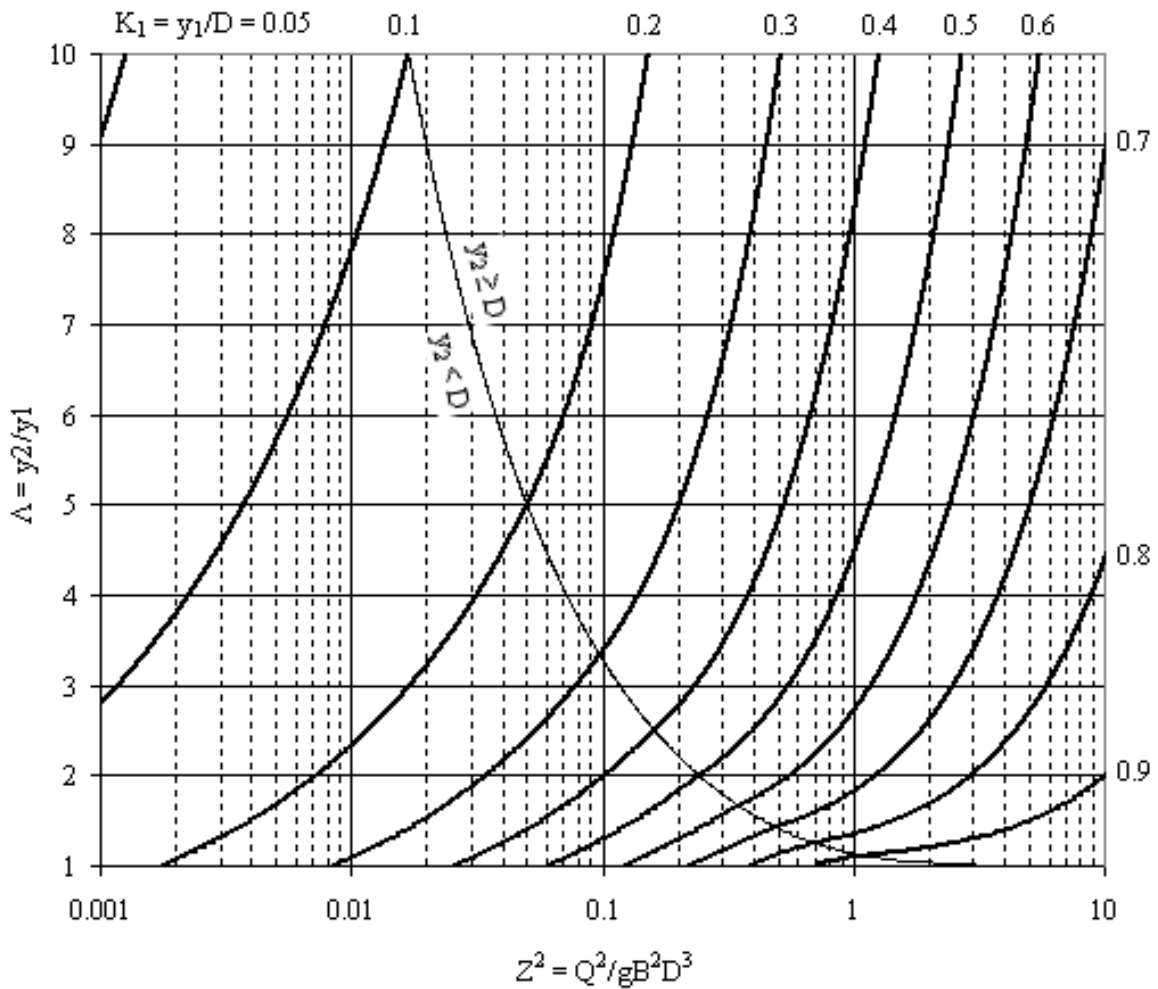


Figure 1. Circular culvert parameters and sequent depth chart

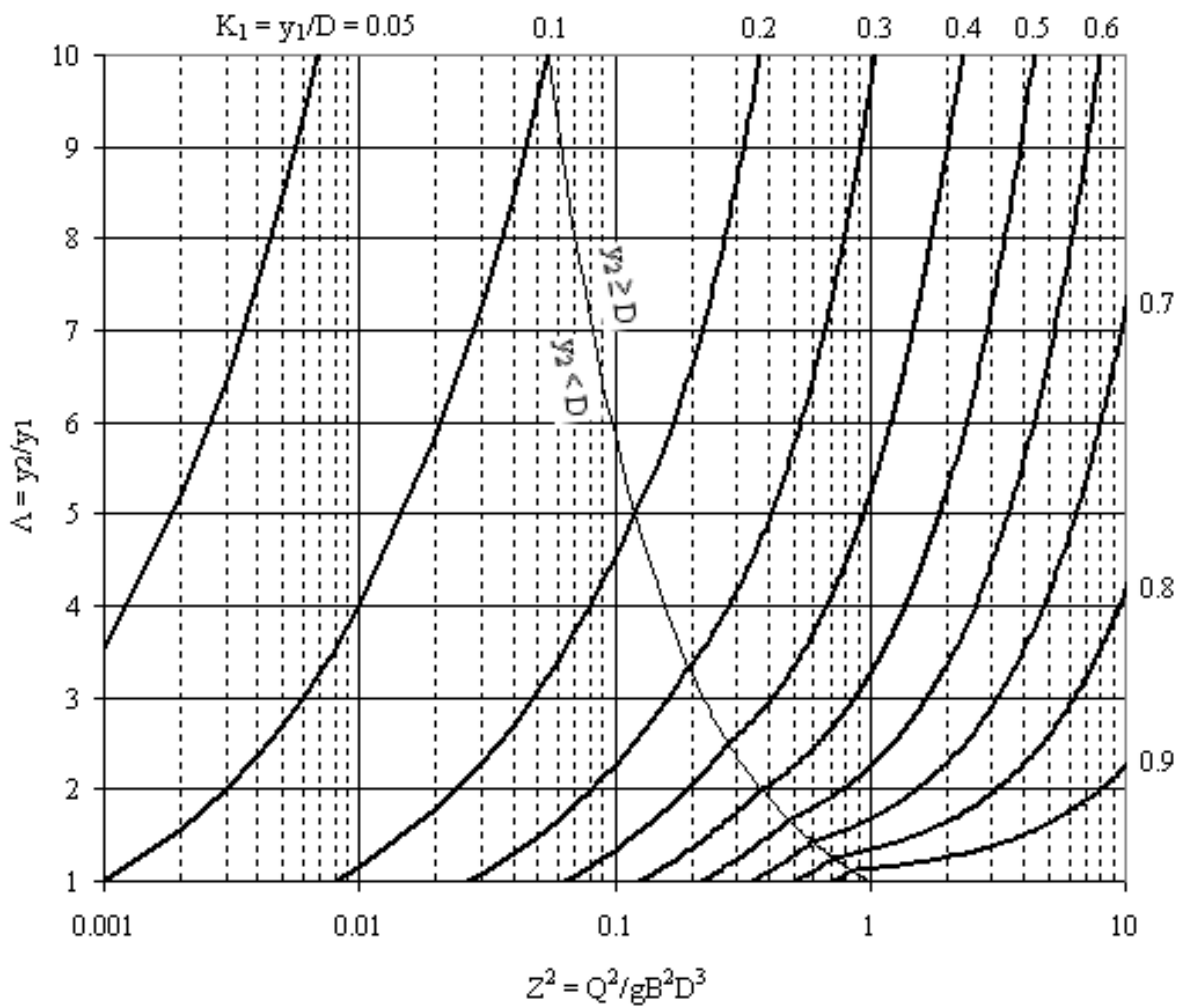
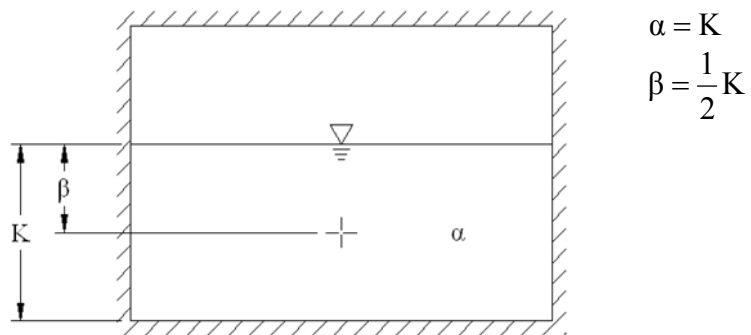
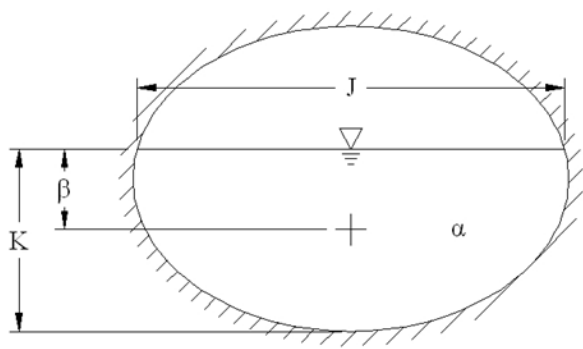


Figure 2. Box culvert parameters and sequent depth chart



$$\alpha = \frac{1}{4} \left\{ J(2K-1) + \tan^{-1} \left( \frac{2K-1}{J} \right) + \frac{\pi}{2} \right\}$$

$$\beta = \frac{1}{4\alpha} \left\{ \frac{1}{2} J(2K-1)^2 + \frac{1}{2} \left[ \tan^{-1} \left( \frac{2K-1}{J} \right) + \frac{\pi}{2} \right] (2K-1) + \frac{J^3}{3} \right\}$$

$$J \equiv \frac{T}{B} = 2\sqrt{K-K^2}$$

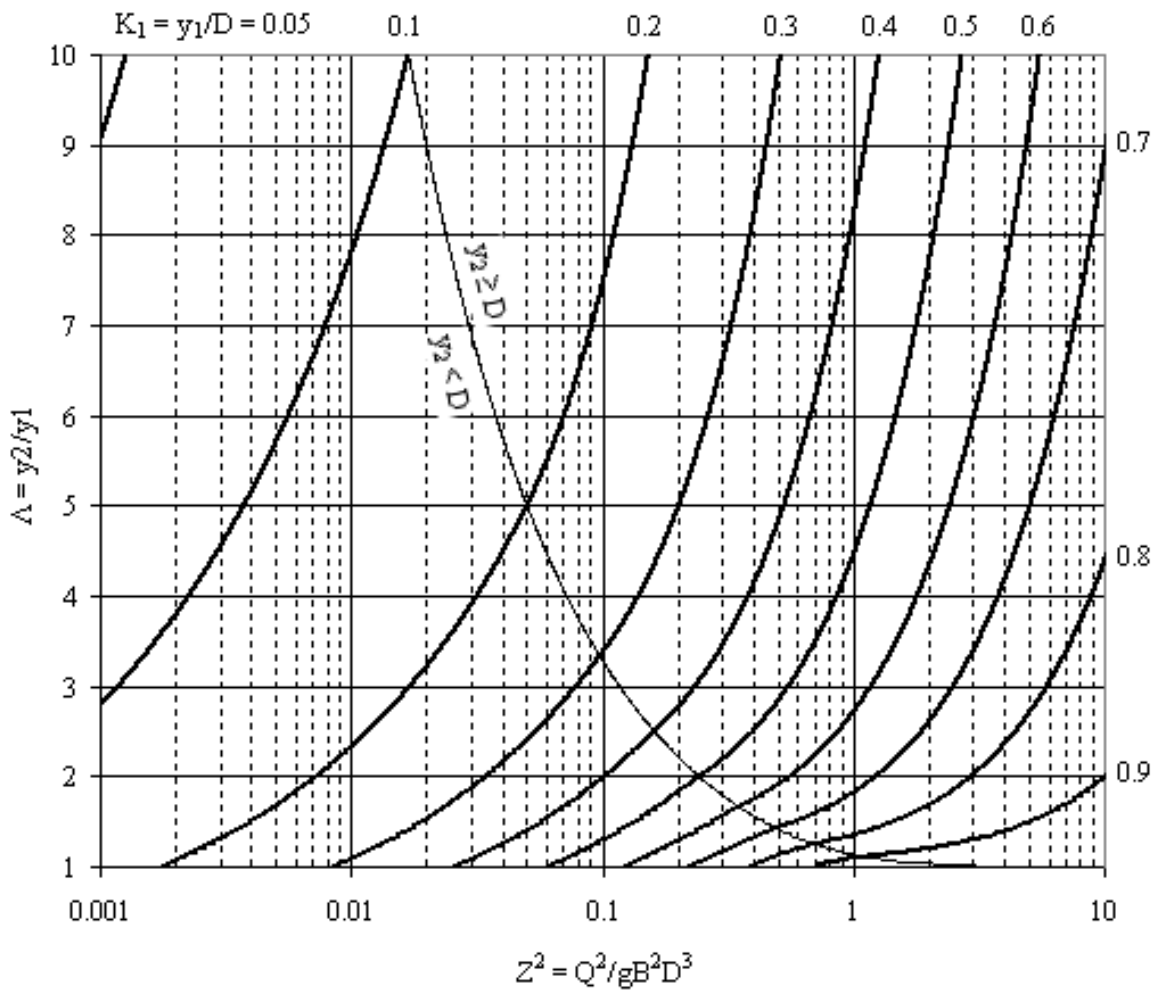
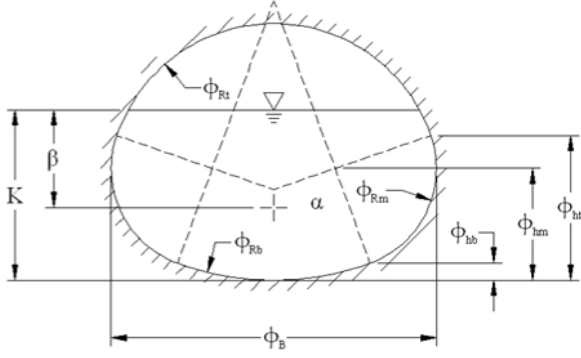


Figure 3. Elliptical culvert parameters and sequent depth chart



$$\alpha = \sum \alpha_i = \begin{cases} \alpha_b|_0^K & \text{for } 0 < K \leq \phi_{hb} \\ \alpha_b|_0^{\phi_{hb}} + \alpha_{m1}|_{\phi_{hb}}^K + \alpha_{m2}|_{\phi_{hb}}^K & \text{for } \phi_{hb} < K \leq \phi_{ht} \\ \alpha_b|_0^{\phi_{hb}} + \alpha_{m1}|_{\phi_{hb}}^{\phi_{hr}} + \alpha_{m2}|_{\phi_{hb}}^{\phi_{hr}} + \alpha_t|_{\phi_{hr}}^K & \text{for } K > \phi_{ht} \end{cases}$$

$$\beta = \frac{\sum \beta_i \alpha_i}{\sum \alpha_i} = \begin{cases} \beta_b|_0^K & \text{for } 0 < K \leq \phi_{hb} \\ \frac{\left( (K - \phi_{hb} + \beta_b|_0^{\phi_{hb}}) \alpha_b|_0^{\phi_{hb}} \right) + \beta_{m1}|_{\phi_{hb}}^K \alpha_{m1}|_{\phi_{hb}}^K + \beta_{m2}|_{\phi_{hb}}^K \alpha_{m2}|_{\phi_{hb}}^K}{\alpha_b|_0^{\phi_{hb}} + \alpha_{m1}|_{\phi_{hb}}^K + \alpha_{m2}|_{\phi_{hb}}^K} & \text{for } \phi_{hb} < K \leq \phi_{ht} \\ \frac{\left( (K - \phi_{hb} + \beta_b|_0^{\phi_{hb}}) \alpha_b|_0^{\phi_{hb}} \right) + (K - \phi_{ht} + \beta_{m1}|_{\phi_{hb}}^{\phi_{ht}}) \alpha_{m1}|_{\phi_{hb}}^{\phi_{ht}} + (K - \phi_{ht} + \beta_{m2}|_{\phi_{hb}}^{\phi_{ht}}) \alpha_{m2}|_{\phi_{hb}}^{\phi_{ht}} + \beta_t|_{\phi_{ht}}^K \alpha_t|_{\phi_{ht}}^K}{\alpha_b|_0^{\phi_{hb}} + \alpha_{m1}|_{\phi_{hb}}^{\phi_{ht}} + \alpha_{m2}|_{\phi_{hb}}^{\phi_{ht}} + \alpha_t|_{\phi_{ht}}^K} & \text{for } K > \phi_{ht} \end{cases}$$

$$\alpha_b = \frac{\phi_{Rb}^2}{2\phi_B} (\theta_b - \sin\theta_b)$$

$$\alpha_{m1} = \frac{1}{\phi_B} (\phi_B - 2\phi_{Rm})(K - \phi_{hm} + \phi_{Rm})$$

$$\alpha_{m2} = \frac{\phi_{Rm}^2}{2\phi_B} (\theta_m - \sin\theta_m)$$

$$\alpha_t = \frac{\phi_{Rt}^2}{2\phi_B} (\theta_t - \sin\theta_t)$$

$$\beta_b = \frac{\phi_{Rb}^3}{3\alpha_b} \left[ 3\sin\left(\frac{\theta_b}{2}\right) - \sin^3\left(\frac{\theta_b}{2}\right) - 3\left(\frac{\theta_b}{2}\right)\cos\left(\frac{\theta_b}{2}\right) \right]$$

$$\beta_{m1} = \frac{1}{2\alpha_{m1}} (\phi_B - 2\phi_{Rm})(K - \phi_{hm} + \phi_{Rm})^2$$

$$\beta_{m2} = \frac{\phi_{Rm}^3}{3\alpha_{m2}} \left[ 3\sin\left(\frac{\theta_m}{2}\right) - \sin^3\left(\frac{\theta_m}{2}\right) - 3\left(\frac{\theta_m}{2}\right)\cos\left(\frac{\theta_m}{2}\right) \right]$$

$$\beta_t = \frac{\phi_{Rt}^3}{3\alpha_t} \left[ 3\sin\left(\frac{\theta_t}{2}\right) - \sin^3\left(\frac{\theta_t}{2}\right) - 3\left(\frac{\theta_t}{2}\right)\cos\left(\frac{\theta_t}{2}\right) \right]$$

$$\theta_b = 2\cos^{-1}\left(1 - \frac{K}{\phi_{Rb}}\right)$$

$$\theta_m = 2\cos^{-1}\left(1 - \frac{1}{\phi_{Rm}}(K - \phi_{hm} + \phi_{Rm})\right)$$

$$\theta_t = 2\cos^{-1}\left(1 - \frac{1}{\phi_{Rt}}(K + 2\phi_{Rt} - 1)\right)$$

$$\phi_{Rb} \equiv \frac{R_b}{D}$$

$$\phi_{hb} \equiv \frac{h_b}{D}$$

$$\phi_{Rm} \equiv \frac{R_m}{D}$$

$$\phi_{hm} \equiv \frac{h_m}{D}$$

$$\phi_{Rt} \equiv \frac{R_t}{D}$$

$$\phi_{ht} \equiv \frac{h_t}{D}$$

$$\phi_B \equiv \frac{B}{D}$$

Figure 4. Pipe arch culvert parameters



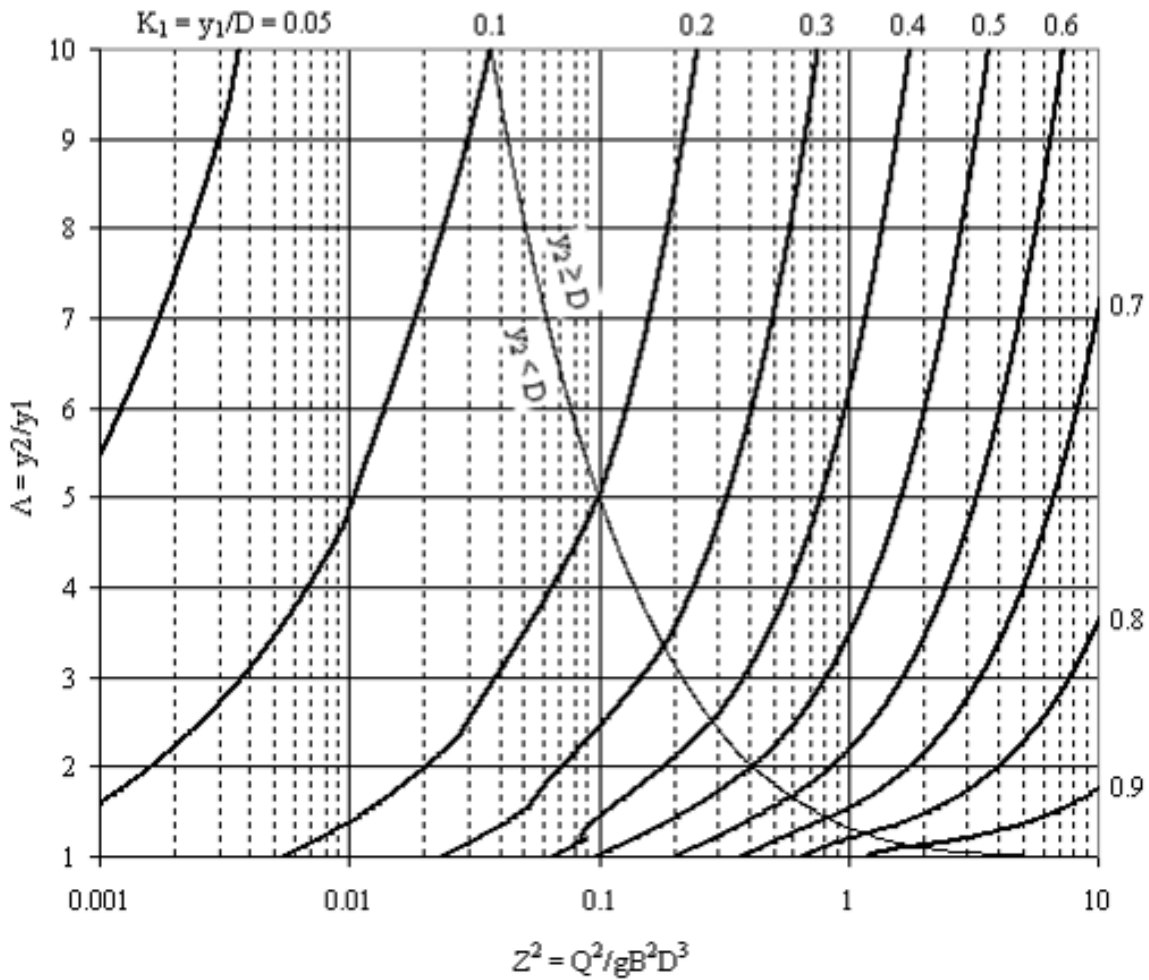


Figure 5. Pipe arch sequent depth chart, based on the following example parameters:  $B = 157$  in,  $D = 100$  in,  $R_b = 100$  in,  $R_m = 300.8$  in,  $R_t = 31.75$  in,  $h_m = 81.67$  in (CONTECH Aluminum Structural Plate)

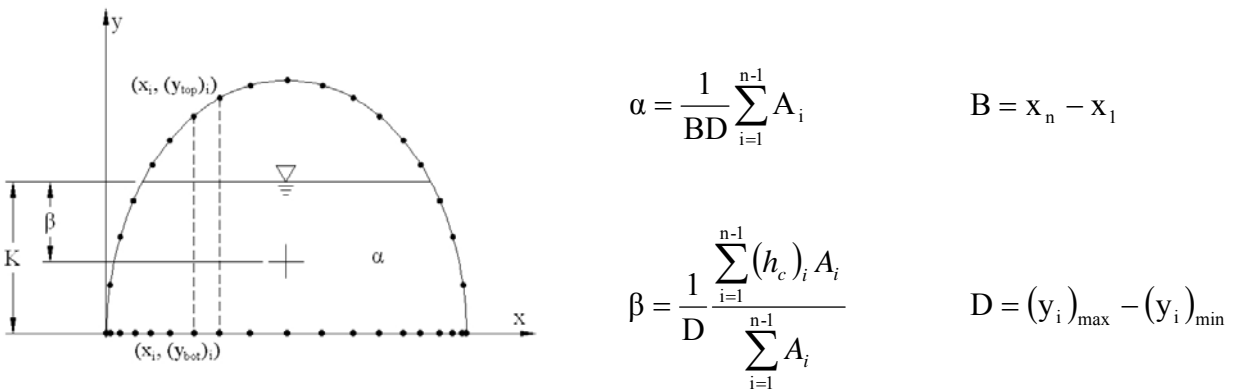


Figure 6. User-defined culvert parameters

Table 2. User-defined culvert parameters (cont'd)

Case	$A_i$	$(h_c)_i$
$0 < y \leq y_A$	0	0
$y_A < y \leq y_B$	$\frac{1}{2} \left( \frac{x_{i+1} - x_i}{y_B - y_A} \right) (y - y_A)^2$	$\frac{1}{3} (y - y_A)$
$y_B < y \leq y_C$	$\frac{1}{2} (x_{i+1} - x_i) [(y - y_B) + (y - y_A)]$	$\frac{1}{4} [(y - y_A) + (y - y_B)]$
$y_C < y \leq y_D$	$\frac{1}{2} (x_{i+1} - x_i) [(y_C - y_B) + (y_D - y_A)] -$ $\frac{1}{2} \left( \frac{x_{i+1} - x_i}{y_D - y_C} \right) (y_D - y)^2$	$\frac{1}{A_i} \left\{ \begin{array}{l} \frac{1}{4} [(y - y_A) + (y - y_B) + (y - y_C) + (y - y_D)] \times \\ \frac{1}{2} (x_{i+1} - x_i) [(y_C - y_B) + (y_D - y_A)] - \\ \frac{1}{3} (y_D - y) \times \frac{1}{2} \left( \frac{x_{i+1} - x_i}{y_D - y_C} \right) (y_D - y)^2 \end{array} \right\}$
$y_D < y \leq D$	$\frac{1}{2} (x_{i+1} - x_i) [(y_C - y_B) + (y_D - y_A)]$	$\frac{1}{4} [(y - y_A) + (y - y_B) + (y - y_C) + (y - y_D)]$

$$y_A = \min((y_{bot})_i, (y_{bot})_{i+1})$$

$$y_B = \max((y_{bot})_i, (y_{bot})_{i+1})$$

$$y_C = \min((y_{top})_i, (y_{top})_{i+1})$$

$$y_D = \max((y_{top})_i, (y_{top})_{i+1})$$

Table 3. Example coordinates for an open-bottom arch, used for Figure 7:

#	X (ft)	Y-Top (ft)	Y-Bottom (ft)
1	0	0.01	0
2	0.22	0.35	0
3	0.47	0.67	0
4	0.76	0.96	0
5	1.08	1.2	0
6	1.43	1.41	0
7	1.81	1.58	0
8	2.19	1.7	0
9	2.59	1.77	0
10	3	1.79	0
11	3.41	1.77	0
12	3.81	1.7	0
13	4.19	1.58	0
14	4.57	1.41	0
15	4.92	1.2	0
16	5.24	0.96	0
17	5.53	0.67	0
18	5.78	0.35	0
19	6	0.01	0

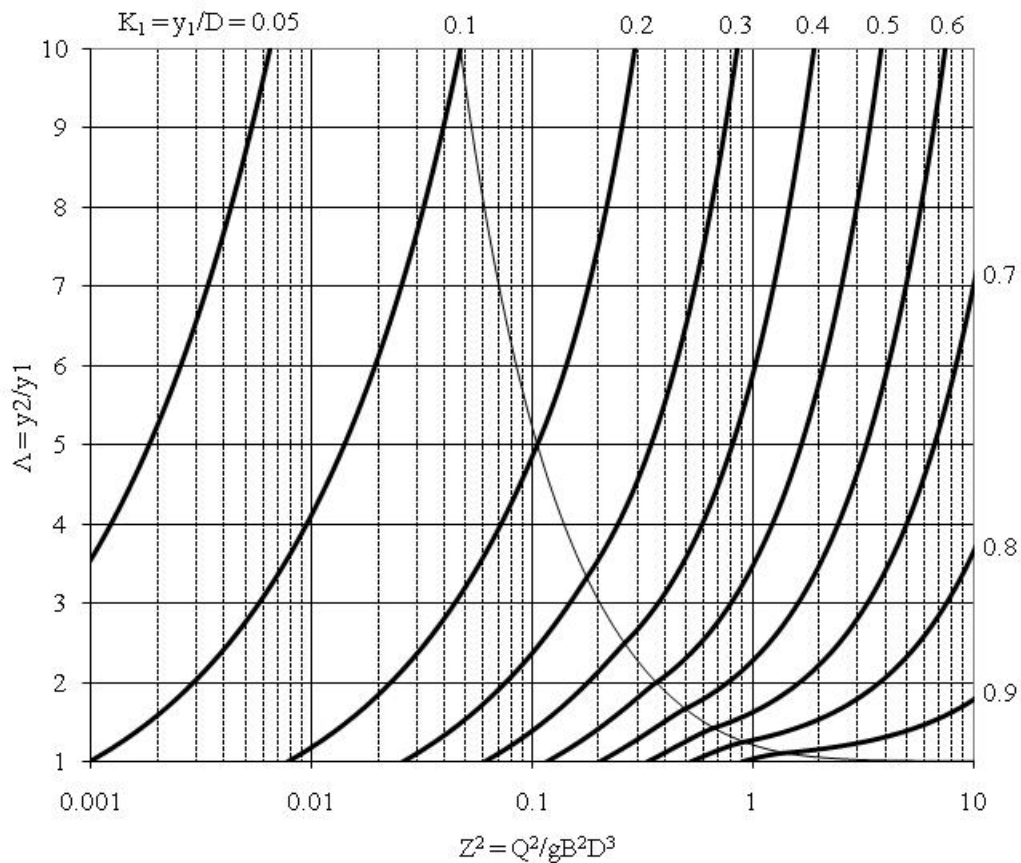
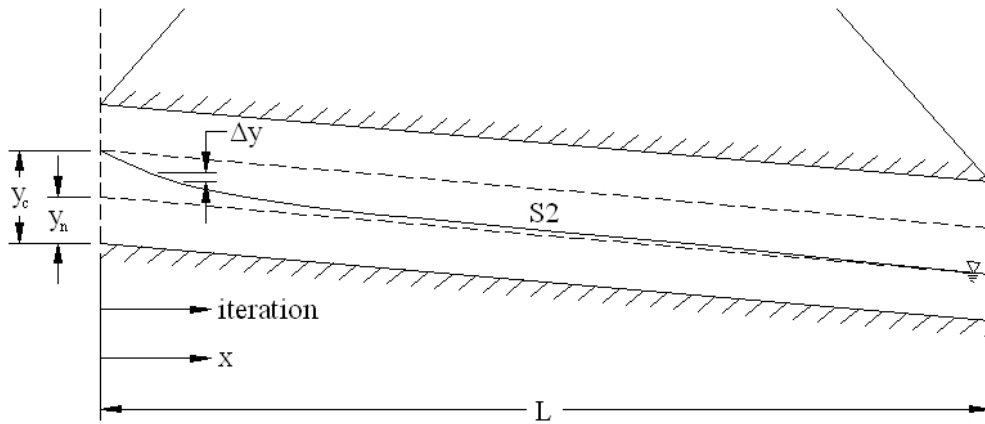


Figure 7. User-defined culvert parameters, based on the example coordinates given in Table 3.

## Hydraulic Jump Profiles in HY-8 Culverts

The following outline presents a possible procedure for adding hydraulic jumps to a Type 1 or 5 flow profile.

1. Calculate S2 curve  $((x_{S2})_i, (y_{S2})_i)$ 
  - a. Start at inlet
    - i.  $(x_{S2})_1 = x_{\text{inlet}}$
    - ii.  $(y_{S2})_1 = y_c$
  - b. Iterate forward with direct step method
    - i. Choose  $\Delta y$
    - ii.  $(y_{S2})_{i+1} = (y_{S2})_i - \Delta y$
    - iii.  $(x_{S2})_{i+1} = f(\text{shape parameters, units, } Q, n, S_0, (y_{S2})_{i+1})$
  - c. Interpolate values at outlet
  - d. Store all  $(x_{S2})_i$  and  $(y_{S2})_i$  values as vectors



2. Calculate S1 curve  $((x_{S1})_i, (y_{S1})_i)$

a. Start at outlet

i.  $(x_{S1})_1 = x_{\text{outlet}}$

ii.  $(y_{S1})_1 = TW$

b. Iterate backward with direct step method

i. Choose  $\Delta y$

ii.  $(y_{S1})_{i+1} = (y_{S1})_i - \Delta y$

iii. If  $(y_{S1})_{i+1} \geq y_c$

a)  $(x_{S1})_{i+1} = f(\text{shape parameters, units, } Q, n, S_0, (y_{S1})_{i+1})$

iv. Else

a)  $(x_{S1})_{i+1} = (x_{S1})_i$

b)  $(y_{S1})_{i+1} = y_c$

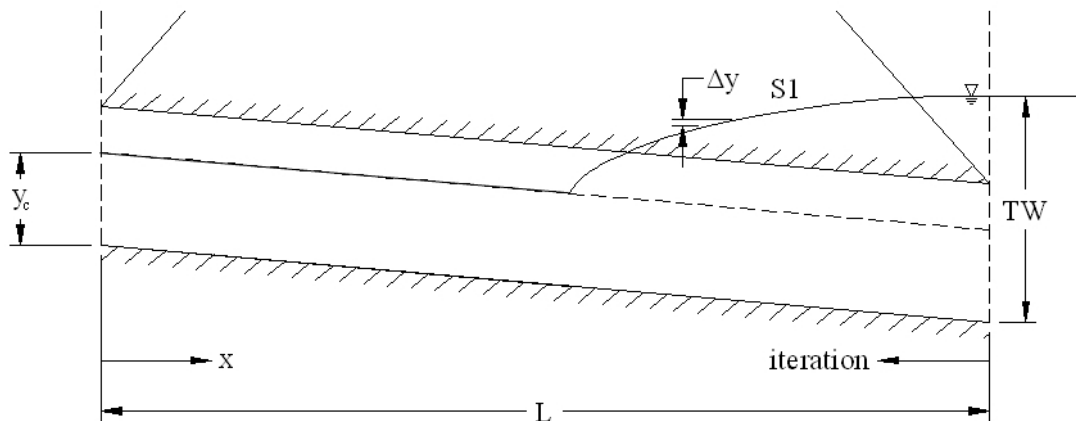
c)  $(x_{S1})_{i+2} = 0$

d)  $(y_{S1})_{i+2} = y_c$

v. If necessary, interpolate values at inlet

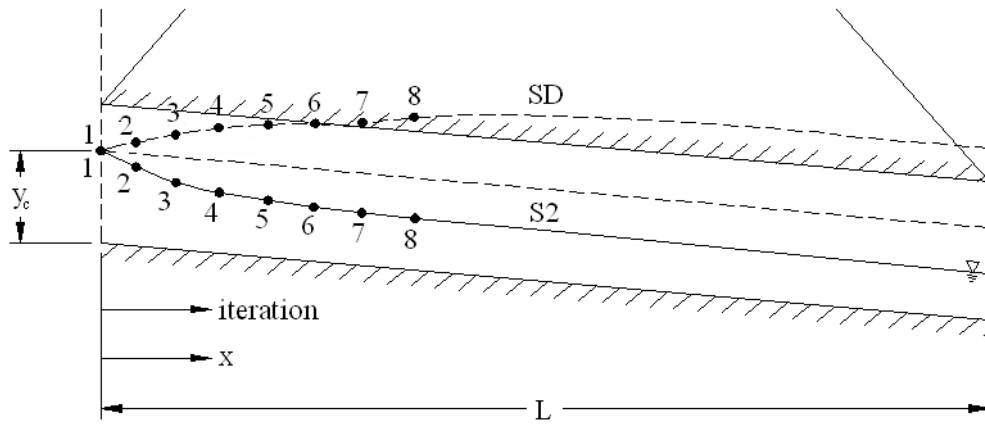
vi. Store all  $(x_{S1})_i$  and  $(y_{S1})_i$  values as vectors

c. Reverse vectors so that  $(x_{S1})_1$  and  $(y_{S1})_1$  correspond to inlet, not outlet



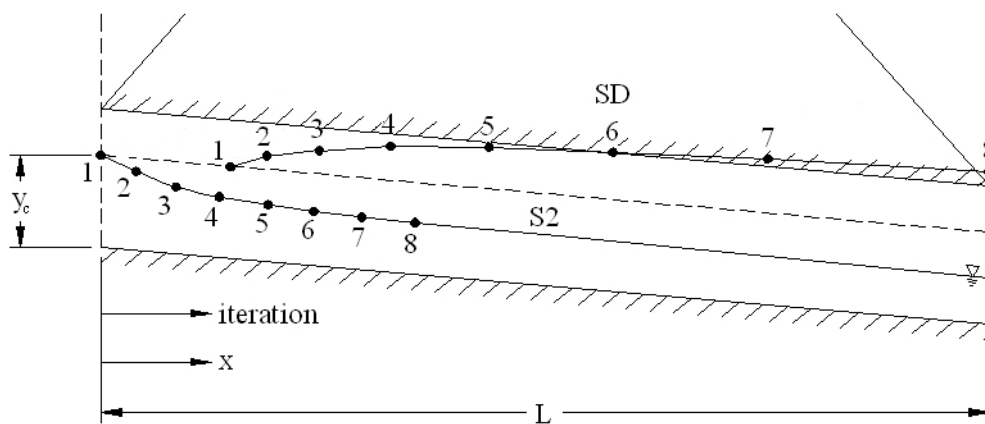
3. Calculate Sequent Depth (SD) curve  $((x_{SD})_i, (y_{SD})_i)$

- a. For each  $(x_{S2})_i$ :
  - i.  $(x_{SD})_i = (x_{S2})_i$
  - ii.  $(y_{SD})_i = f(\text{shape parameters}, Q, (y_{S2})_i)$
- b. Store all  $(x_{SD})_i$  and  $(y_{SD})_i$  values as vectors

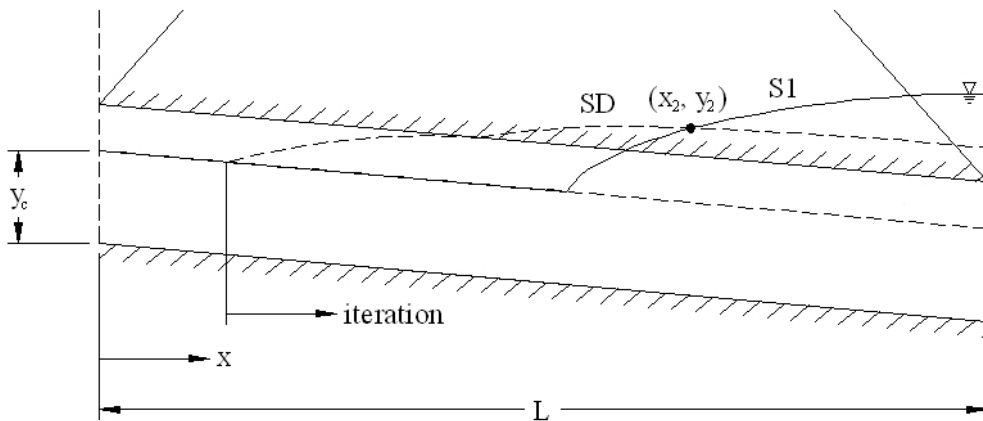


4. Shift and truncate SD curve to account for jump length, as per Chow 1959, p. 399  $((x_{SD'})_i, (y_{SD'})_i)$

- a. For each  $(x_{SD})_i$ :
  - i.  $(L_j)_i = f(\text{shape parameters}, (y_{S2})_i, (y_{SD})_i)$
  - ii.  $(x_{SD'})_i = (x_{SD})_i + (L_j)_i$
- b. Store all  $(x_{SD'})_i$  and  $(y_{SD'})_i$  values as vectors

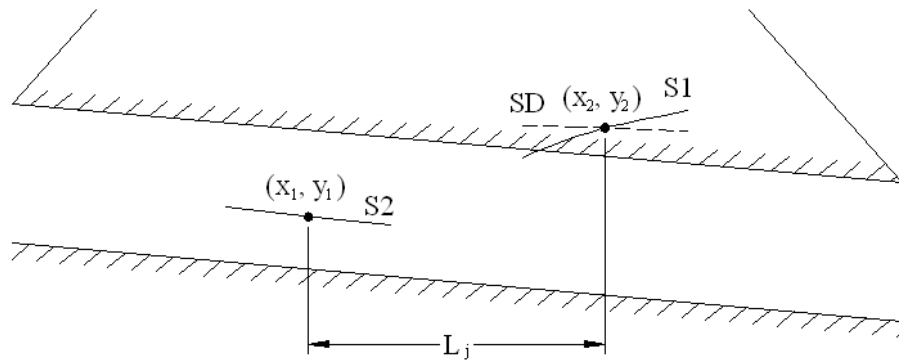


5. Compare shifted SD and S1 curves to find occurrence of hydraulic jump  $(x_1, y_1), (x_2, y_2)$ 
  - a. Determine whether jump will occur at all
    - i. Interpolate  $y_{S1}$  at  $(x_{SD'})_1$
    - ii. If  $y_{S1} > (y_{SD'})_1$ 
      - a) A jump will NOT occur
      - b) Plot S1 curve only
      - c) Find HW from S1 curve
    - iii. Otherwise
      - a) A jump MAY occur
      - b) Continue to Step 5.b
  - b. Iterate through shifted SD points and compare with S1 curve to find  $(x_2, y_2)$ 
    - i. Interpolate  $(y_{S1})_i$  at  $(x_{SD'})_i$  and  $(y_{S1})_{i+1}$  at  $(x_{SD'})_{i+1}$
    - ii. If  $(y_{S1})_i \leq (y_{SD'})_i$  and  $(y_{S1})_{i+1} > (y_{SD'})_i$ 
      - a) A jump WILL occur
      - b) Interpolate to find  $(x_2, y_2)$  at intersection of S1 and SD' curves
      - c) Continue to Step 5.d



- iii. Otherwise
  - a) A jump MAY occur further on
  - b) Move on to next SD' point and repeat Steps 5.b.i through 5.b.iii

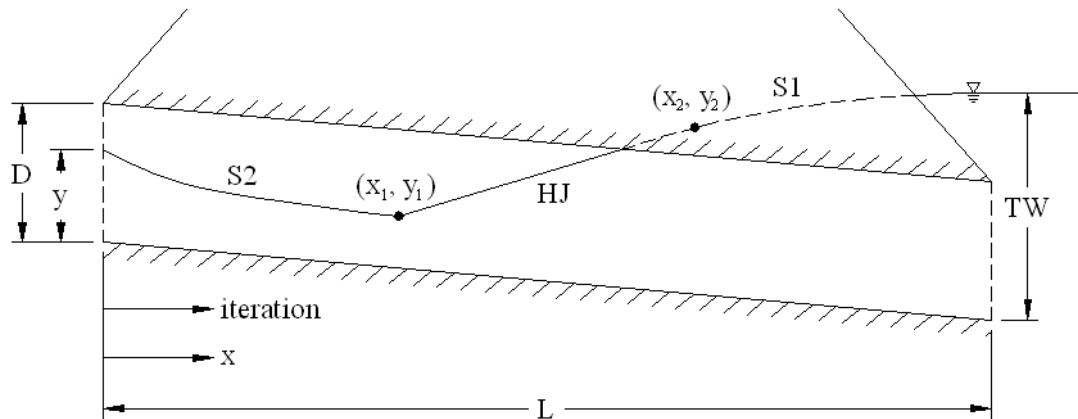
- c. If by the last shifted SD point the two curves haven't intersected,
  - i. A jump will NOT occur
  - ii. Assume jump is completely swept out of culvert
  - iii. Plot S2 curve only
- d. Back-calculate  $(x_1, y_1)$  for a hydraulic jump
  - i. Iterate to find  $x_1, y_1,$  and  $L_j$ 
    - a) Choose  $y_1$
    - b)  $L_j = f(\text{shape parameters}, y_1, y_2)$
    - c)  $x_1 = x_2 - L_j$
    - d) Interpolate new  $y_1$  at  $x_1$  from S2 curve
    - e) Repeat Steps 5.d.i.b through 5.d.i.d until some error criteria is met





6. Plot profile

- a. For  $x = x_{inlet}$  to  $x_1$ , plot S2 curve
- b. For  $x = x_1$  to  $x_2$ , plot hydraulic jump (HJ)
- c. For  $x = x_2$  to  $x_{outlet}$ , plot S1 curve



The following figures demonstrate the results of Visual Basic code written to test this logic. In this test, 7.5 cfs was passed through a 2-foot-diameter circular culvert at a 5% slope, with a Manning's roughness value of 0.013. The tailwater was adjusted to test each of the possible outcomes of the algorithm.

