State Planning and Research Program
Quarterly Report

PROJECT TITLE: Design and Construction Guidelines for Thermally Insulated Concrete Pavements

OBJECTIVES:
The main objective of the proposed research is to develop design and construction guidelines for thermally insulated concrete pavements (TICP), i.e. composite thin HMA overlays of new or structurally sound existing PCC pavements. A secondary objective is to develop recommendations for feasibility analysis of newly constructed TICP or thin overlays of the existing concrete pavements.

PERIOD COVERED: January – March 31, 2010

PARTICIPATING AGENCIES: Minnesota Department Of Transportation, Caltrans, Federal Highway Administration, Local Road Research Board, Washington State Department of Transportation

PROJECT MANAGER: Tim Clyne **LEAD AGENCY**: Minnesota Dept. of Transportation **PRINCIPAL INVESTIGATOR**: Lev Khazanovich	**SP&R PROJECT NO**: TPF 5(149)	PROJECT IS: ______ Planning __X__ Research & Development
ANNUAL BUDGET: The total project budget is $455,000. Of that $16,000 is reserved for pooled fund administrative costs, which leaves $439,000 available for research.	**PROJECT EXPENDITURES TO DATE**: The estimated expenses are $165,400.	

WORK COMPLETED:
See attached.

SUMMARY OF ACTIVITIES EXPECTED TO BE PERFORMED NEXT QUARTER:
The research team will finalize validation of the MEPDG EICM model and will continue work on improvements to the MEPDG structural models.

STATUS AND COMPLETION DATE: Due to delay bringing in issuing subcontact/consulting agreements with the Dr, John Harvey and other members of the research team, there will be a need for a no-cost time extension of the project. The request will be submitted in May 2010.

.

Task 3. EICM Validation and Analysis

Sensitivity analysis of the effect of EICM predictions on the overall MEPDG performance prediction was conducted. This work also included an analysis of the effect of the thermal conductivity, heat capacity, and the concrete coefficient of thermal expansion in both an AC/PCC and PCC pavement. Both pavement structures used the Minneapolis – St. Paul, MN, EICM file as the climate input. Different traffic levels were selected for the two pavement structures. This was done so the predicted percentage of cracked slabs would be 20% when using default MEPDG inputs. Average Annual Daily Truck Traffic (AADTT) levels were set at 7420 for the AC/PCC structure, and 8250 for the PCC structure. The parameters under consideration, thermal conductivity and heat capacity, were adjusted +/- 25% from the default values in the MEPDG version 1.1.

Heat Capacity

When the heat capacity was increased, the predicted amount of transverse cracking in the PCC layer was reduced. This was true for the AC & PCC layers in the composite structure and the PCC layer in the rigid structure. A change in heat capacity in the AC layer of the composite pavement had the smallest effect on predicted transverse cracking. The PCC-only rigid pavement was most sensitive to a change in heat capacity. The results are listed in the table below.

Heat Capacity – AC	**% Cracking AC/PCC Pavement**	**% Cracking PCC Pavement**
0.1725	25.8	-
0.23 (default)	20	-
0.2875	15.2	-
Heat Capacity – PCC	**% Cracking AC/PCC Pavement**	**% Cracking PCC Pavement**
0.21	32.1	45.3
0.28 (default)	20	20
0.35	12.2	10.7

Thermal Conductivity

Thermal conductivity in the AC and PCC layers was also examined. An increase in thermal conductivity in the AC layer resulted in higher predicted cracking in the PCC layer. However, an increase in thermal conductivity in the PCC layer resulted in lower predicted cracking. At first glance, this may seem contradictory, but it is not. A temperature gradient in the PCC layer will induce internal stresses. When the conductivity of the AC layer is increased, more heat conducts from the AC layer to the top of the PCC layer. Therefore, the top of the PCC layer is warmer than the bottom, resulting in a thermal gradient and thermal stresses. Conversely, if the conductivity of the AC layer is lower, then not as much heat will be transferred to the PCC layer, resulting in lower thermal gradients, and lower thermal stresses. An increase in thermal conductivity in the PCC layer would reduce thermal gradients in the PCC layer because heat will transfer easier. Ideally, an AC layer with low thermal conductivity and PCC layer with high thermal conductivity would reduce thermal gradients in the PCC layer, thereby reducing thermal stresses. The results are listed in the table below.

Thermal Conductivity AC	% Cracking AC/PCC Pavement	% Cracking PCC Pavement
0.5025	7.7	-
0.67 (default)	20	-
0.8375	33	-
Thermal Conductivity PCC	**% Cracking AC/PCC Pavement**	**% Cracking PCC Pavement**
0.9375	35.6	35.1
1.25 (default)	20	20
1.5625	11.2	15.1

Coefficient of Thermal Expansion

The sensitivity to the coefficient of thermal expansion of the PCC layer was also examined, and the results are listed below.

Coefficient of Thermal Expansion of the PCC Layer	% Cracking AC/PCC Pavement	% Cracking PCC Pavement
4.125	13.3	3.3
5.5	20	20
6.875	30.7	78.7

As expected, a decrease in the coefficient of thermal expansion of the PCC layer resulted in lower predicted transverse cracking values from the MEPDG. It is important to note that the composite AC/PCC structure was far less sensitive than the PCC-only rigid pavement to differences in the coefficient of thermal expansion of the PCC layer. This appears to be due to the insulating effect of the AC layer. When the PCC layer is not insulated it is subjected to greater temperature fluctuations and temperature extremes, both of which are exacerbated by an increased coefficient of thermal expansion and thus result in higher cracking.

Task 4. Evaluation of Pavement Response Models

Last quarter, the following activities were conducted:

- Development of a computational procedure for the non-linear strain-causing temperature stresses using equivalency concepts for a 3-layered system of AC – PCC – Base, and
- Development of rapid solutions for predicting critical PCC bottom surface stresses.

Computation of the non-linear strain-causing temperature stresses using equivalency concepts for 3-layered system of AC – PCC – Base.

In order to evaluate the stresses in the PCC layer of an AC over PCC pavement, the methodology adopted by the Mechanistic Empirical Pavement Design Guide (MEPDG) is followed closely in this research and the document. The solutions developed for a two-layered system of PCC-Base are extended to the three-layered system of AC-PCC-Base. The analysis involves the equivalent single layer slab concept which states that PCC stresses in a three-layered slab can be found from the corresponding stresses in the equivalent homogeneous plate that exhibits the same deflection profile as the in situ pavement (Ioannides et al. 1992).

Equivalent Linear Temperature Distribution Concept:

The equivalent temperature gradient concept for a single-layer slab was introduced by Thomlinson (1940) and was further developed by other researchers (Choubane and Tia 1992, Mohamed and Hansen 1997). The concept was later generalized for a non-uniform, multi-layered slab (Khazanovich 1994, Ioannides and Khazanovich 1998). This concept states that if two slabs have the same plane-view geometry, flexural stiffness, self-weight, boundary conditions, and applied pressure, and rest on the same foundation, then these slabs have the same deflection and bending moments distributions if their through-the-thickness temperature distributions satisfy the following condition:

$$\int_{h_a} E_a(z)\alpha_a(z)(T_a(z) - T_{0,a})zdz = \int_{h_b} E_b(z)\alpha_b(z)(T_b(z) - T_{0,b})zdz \qquad (1)$$

where: a and b are subscripts denoting two slabs,
z is the distance from the neutral axis,
T_0 is the temperatures at which theses slabs are assumed to be flat,
$T(z)$ is the temperature distribution in the slab,
α is the coefficient of thermal expansion,
E is the modulus of elasticity, and
h is the slab thickness.

To apply this concept for the curling analysis of a three-layered system, the temperature distribution throughout the three-layered slab thickness should be split into its three components:

- The temperature component that causes constant strain throughout-the-slab-thickness.
- The temperature component that causes linear strain throughout-the-slab-thickness.
- The temperature component that causes nonlinear strain.

The constant strain-causing temperature component $T_c(z)$, in its general form, is given as:

$$T_c(z) = T_o + \frac{\sum_{i=1}^{l} \int_h \alpha(z)E(z)\left[T(z) - T_o\right]dz}{\alpha(z)\sum_{i=1}^{l} \int_h E(z)dz} \qquad (2)$$

where: i is the layer index, and
l is the total no. of layers in the system.

The linear strain-causing temperature component $T_L(z)$, in its general form, is given as:

$$T_L(z) = T_o + \frac{z}{\alpha(z)} \frac{\sum_{i=1}^{l} \int_h \alpha(z)E(z)[T(z) - T_o] z dz}{\sum_{i=1}^{l} \int_h E(z) z^2 dz} \tag{3}$$

By definition, the total temperature distribution is given as:

$$T(z) - T_o = [T_c(z) - T_o] + [T_L(z) - T_o] + [T_{NL}(z) - T_o] \tag{4}$$

Therefore, the nonlinear strain-causing temperature component $T_{NL}(z)$ is:

$$T_{NL}(z) - T_o = T(z) - [T_c(z) - T_o] - [T_L(z) - T_o] - T_o \tag{5}$$

The corresponding nonlinear temperature components and stresses at the bottom and the top of the PCC layer are given as:

$$(T_{NL,PCC,bot} - T_o) = T_{11} - (T_{c,PCC} - T_o) - (T_{L,PCC,bot} - T_o) - T_o \tag{6}$$

$$\sigma_{NL,PCC,bot} = -\frac{E_{PCC}\alpha_{PCC}}{(1-\mu)}\left(T_{NL,PCC,bot} - T_o\right) \tag{7}$$

$$(T_{NL,PCC,top} - T_o) = T_1 - (T_{c,PCC} - T_o) - (T_{L,PCC,top} - T_o) - T_o \tag{8}$$

$$\sigma_{NL,PCC,top} = -\frac{E_{PCC}\alpha_{PCC}}{(1-\mu)}\left(T_{NL,PCC,top} - T_o\right) \tag{9}$$

where: σ_{NL} is the stress due to nonlinear temperature component,
μ is the Poisson's ratio for PCC,
T_1 is the temperature at the top surface of the PCC layer, and
T_{11} is the temperature at the bottom surface of the PCC layer

The following analysis details the process of computing constant-, linear-, and non-linear strain-causing temperature components for a case of fully bonded AC/PCC and fully bonded PCC/base interfaces. There are three (3) other cases namely,

1. Unbonded AC/PCC and unbonded PCC/base interfaces,
2. Unbonded AC/PCC and fully bonded PCC/base interfaces, and
3. Fully bonded AC/PCC and unbonded PCC/base interfaces.

The equivalency analysis for these cases is documented in Appendix A.

<u>The Equivalency Analysis for Fully Bonded AC/PCC and Fully Bonded PCC/Base Interfaces:</u>

For a fully bonded system of AC-PCC-base layers, the distance between the neutral axis (N.A.) and the top surface of the AC layer is determined from the following equation:

$$x = \frac{\frac{E_{AC}}{E_{PCC}}\frac{h_{AC}^2}{2} + h_{PCC}\left(h_{AC} + \frac{h_{PCC}}{2}\right) + \frac{E_{Base}}{E_{PCC}}h_{Base}\left(h_{AC} + h_{PCC} + \frac{h_{Base}}{2}\right)}{\frac{E_{AC}}{E_{PCC}}h_{AC} + h_{PCC} + \frac{E_{Base}}{E_{PCC}}h_{Base}} \tag{10}$$

where, x is the depth of the N.A. from the top of the AC layer. For an effective single layer slab with the same modulus of elasticity and Poisson's ratio as the PCC layer, the slab thickness and unit weight are given as:

$$h_{eff} = \sqrt[3]{\frac{E_{AC}}{E_{PCC}}h_{AC}^3 + h_{PCC}^3 + \frac{E_{Base}}{E_{PCC}}h_{Base}^3 + 12\left[\frac{E_{AC}}{E_{PCC}}h_{AC}\left(x - \frac{h_{AC}}{2}\right)^2 + h_{PCC}\left(h_{AC} + \frac{h_{PCC}}{2} - x\right)^2 + \frac{E_{Base}}{E_{PCC}}h_{Base}\left(h_{AC} + h_{PCC} + \frac{h_{Base}}{2} - x\right)^2\right]} \tag{11}$$

$$\gamma_{eff} = \frac{h_{AC}\gamma_{AC} + h_{PCC}\gamma_{PCC} + h_{Base}\gamma_{Base}}{h_{eff}} \tag{12}$$

Where h_{eff} is the thickness of the effective single layer slab, and γ is the unit weight. Therefore, the constant strain-causing temperature component derived from eqn. (2) is:

$$T_c(z) - T_o = \frac{1}{\alpha(z)} \frac{\int_{-x}^{h_{AC}-x}\alpha_{AC}E_{AC}[T(z) - T_o]dz + \int_{h_{AC}-x}^{h_{AC}+h_{PCC}-x}\alpha_{PCC}E_{PCC}[T(z) - T_o]dz + \int_{h_{AC}+h_{PCC}-x}^{h_{AC}+h_{PCC}+h_{Base}-x}\alpha_{Base}E_{Base}[T(z) - T_o]dz}{E_{AC}h_{AC} + E_{PCC}h_{PCC} + E_{Base}h_{Base}} \tag{13}$$

In order to maintain consistency with the analysis in MEPDG, it was assumed that

1. The coefficient of thermal expansion of the PCC layer is equal to the coefficient of thermal expansion of the base layer,
2. The temperature T_0 is equal to the temperature at the bottom surface of the PCC slab, and
3. The temperature throughout the base layer is equal to the temperature at the bottom surface of the PCC slab.

Since the temperature distribution in the AC layer is known at 5 points and in the PCC layer is known at 11 points, integrals in eqn. (13) were evaluated numerically resulting in the following:

$$T_c(z) - T_o = \frac{\frac{\alpha_{AC}E_{AC}}{\alpha_{PCC}}\left(\frac{h_{AC}}{8}\left(T_{AC1} + 2*\sum_{i=2}^{4}T_{ACi} + T_{AC5}\right) - T_o h_{AC}\right) + E_{PCC}\left(\frac{h_{PCC}}{20}\left(T_1 + 2*\sum_{i=2}^{10}T_i + T_{11}\right) - T_o h_{PCC}\right)}{E_{AC}h_{AC} + E_{PCC}h_{PCC} + E_{Base}h_{Base}} \tag{14}$$

The linear strain-causing temperature component derived from eqn. (3) is:

$$T_L(z) = T_o + \frac{12z}{\alpha(z)} \frac{\int_{-x}^{h_{AC}-x} \alpha_{AC} E_{AC} [T(z) - T_o] zdz + \int_{h_{AC}-x}^{h_{AC}+h_{PCC}-x} \alpha_{PCC} E_{PCC} [T(z) - T_o] zdz + \int_{h_{AC}+h_{PCC}-x}^{h_{AC}+h_{PCC}+h_{Base}-x} \alpha_{Base} E_{Base} [T(z) - T_o] zdz}{E_{AC} h_{AC}^3 + E_{PCC} h_{PCC}^3 + E_{Base} h_{Base}^3 + 12\left[E_{AC} h_{AC} \left(x - \frac{h_{AC}}{2} \right)^2 + E_{PCC} h_{PCC} \left(h_{AC} + \frac{h_{PCC}}{2} - x \right)^2 + E_{Base} h_{Base} \left(h_{AC} + h_{PCC} + \frac{h_{Base}}{2} - x \right)^2 \right]} \tag{15}$$

The linear temperature distribution in the effective slab is defined by the difference in the temperatures at the top and bottom surfaces such that it causes the same bending moment distributions in the effective single-layer slab as in the original composite slab. This difference is given as:

$$\Delta(T_{L,eff} - T_o) = \frac{-12 h_{eff}}{\alpha_{PCC}} \left(\frac{\int_{-x}^{h_{AC}-x} \alpha_{AC} E_{AC} [T(z) - T_o] zdz + \int_{h_{AC}-x}^{h_{AC}+h_{PCC}-x} \alpha_{PCC} E_{PCC} [T(z) - T_o] zdz + \int_{h_{AC}+h_{PCC}-x}^{h_{AC}+h_{PCC}+h_{Base}-x} \alpha_{Base} E_{Base} [T(z) - T_o] zdz}{E_{PCC} h_{eff}^3} \right) \tag{16}$$

As it was assumed that the temperature in the base layer is constant and equal to the temperature at the bottom of the PCC layer, we have:

$$T_{Base}(z) = T_{11} = T_o$$

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2} \left(\frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \int_{-x}^{h_{AC}-x} T(z) zdz - \frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} T_{11} \int_{-x}^{h_{AC}-x} zdz + \int_{h_{AC}-x}^{h_{AC}+h_{PCC}-x} T(z) zdz - T_{11} \int_{h_{AC}-x}^{h_{AC}+h_{PCC}-x} zdz \right) \tag{17}$$

It can be approximated numerically as:

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2} \left(\begin{array}{l} \frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \frac{h_{AC}}{24} \sum_{i=1}^{4} \left(T_i * \left((3i-2) * \frac{h_{AC}}{4} - 3x \right) + T_{i+1} * \left((3i-1) * \frac{h_{AC}}{4} - 3x \right) \right) \\ - \frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \frac{T_{11}}{2} h_{AC} (h_{AC} - 2x) \\ + \frac{h_{PCC}}{60} \sum_{i=1}^{10} \left(T_i * \left((3i-2) * \frac{h_{PCC}}{10} - 3(x - h_{AC}) \right) + T_{i+1} * \left((3i-1) * \frac{h_{PCC}}{10} - 3(x - h_{AC}) \right) \right) \\ - \frac{T_{11}}{2} h_{PCC} (h_{PCC} + 2h_{AC} - 2x) \end{array} \right) \tag{18}$$

Therefore, the linear strain-causing temperature at the bottom and the top of the PCC layer is given as:

$$T_{L,PCC,bot} - T_o = \frac{\Delta(T_{L,eff} - T_o)}{h_{eff}}(h_{PCC} + h_{AC} - x) \tag{19}$$

$$T_{L,PCC,top} - T_o = -\frac{\Delta(T_{L,eff} - T_o)}{h_{eff}}(x - h_{AC}) \tag{20}$$

Eqns. (14) and (19) along with eqns. (6) and (7) will compute the stress due to non-linear strain-causing temperature at the bottom of the PCC layer and eqns. (14) and (20) along with eqns. (8) and (9) will give the stress at the top of the PCC layer.

Development of rapid solutions for predicting critical PCC bottom surface stresses.

The trained neural networks (NN) as implemented in the MEPDG were used to compute PCC stresses. The stresses were determined for a wide range of site conditions, design parameters, and axle loading. The detailed procedure (AASHTO 2010) described below, was computed for each hour of the pavement design life.

Step 1. Calculate the Effective Slab Thickness

Based on the interface conditions between AC-PCC layers and PCC-Base layers, the effective single-slab thickness was calculated as described in eqn. (11) (for fully bonded interfaces; refer to Appendix A for other cases).

Step 2. Calculate Unit Weight of the Effective Slab

Based on the interface conditions between PCC and base layers, the effective single-slab unit weight was calculated as described in eqn. (12) (for fully bonded interfaces; refer to Appendix A for other cases). The weight of the AC layer is always accounted for as it is over the PCC layer.

Step 3. Calculate Radius of Relative Stiffness

The radius of relative stiffness of the effective slab is:

$$l = \sqrt[4]{\frac{E_{PCC} h_{eff}^3}{12 * (1 - \mu_{eff}^2) * k}} \tag{21}$$

where l is the radius of relative stiffness, and k is the coefficient of subgrade reaction.

Step 4. Calculate Effective Temperature Differential

Based on the interface conditions between AC-PCC layers and PCC-Base layers, the equivalent temperature difference is determined from eqn. (18) (for fully bonded interfaces; refer to Appendix A for other cases).

Step 5. Compute Korenev's Non-dimensional Temperature Gradient

The Korenev's non-dimensional temperature gradient for effective slab is given as:

$$\phi = \frac{2\alpha_{PCC}(1+\mu_{PCC})l^2}{h_{eff}^2}\frac{k}{\gamma_{eff}}\Delta(T_{eff} - T_0) \quad (22)$$

where φ is the non-dimensional temperature gradient.

Step 6. Compute Adjusted Load/Pavement Weigh Ratio (Normalized Load)

$$q^* = \frac{P}{LW\gamma_{eff}h_{eff}} \quad (23)$$

where q^* is the adjusted load/pavement weigh ratio, P is the axle weight, L is the slab length, and W is the Slab width.

Step 7. Calculate Equivalent Slab Thickness

The equivalent slab thickness is a thickness of a slab with the modulus of elasticity and Possion's ratio equal to 4,000,000 psi and 0.15, respectively, resting on the Winkler foundation with the coefficient of subgrade reaction equal to 100 psi/in, and having the same radius of relative stiffness as the effective slab. The equivalent slab thickness is determined using the following equation:

$$h_{eq} = \sqrt[3]{\frac{l^4}{3410}} \quad (24)$$

where h_{eq} is the equivalent slab thickness

Step 8. Compute Curling-Related Stresses in the Equivalent Slab

The NNs were used to compute stresses in the equivalent slab which has the same ratio of radius of relative stiffness to joint spacing, joint spacing, traffic offset and appropriate Korenev's non-dimensional temperature gradient, φ, and normalized load ratio q^*. When the pavement is loaded by a single axle load, neural network NNA1 was employed. For tandem or tridem axle loads NNA2 was used. The following cases were considered:

- Case I – resulting stress $\sigma_{eq}^{A}(P,\Delta T)$: Korenev's non-dimensional temperature gradient, φ, is equal to the nondimensional temperature gradient determined in Step 5; normalized load ratio q^* is equal to normalized load ratio determined in Step 6.
- Case II – resulting stress $\sigma_{eq}^{A}(0,\Delta T)$: Korenev's non-dimensional temperature gradient, φ, is equal to the nondimensional temperature gradient determined in Step 5; normalized load ratio q^* is equal 0.
- Case III – resulting stress $\sigma_{eq}^{A}(P,0)$: Korenev's non-dimensional temperature gradient, φ, is equal to 0; normalized load ratio q^* is equal to normalized load ratio determined in Step 6.

<u>Step 9. Compute Curling-Related Stresses in the Effective Slab</u>

The stresses obtained in step 8 represent stresses in the equivalent slab with the modulus of elasticity and Possion's ratio equal to 4,000,000 psi and 0.15, respectively, resting on the Winkler foundation with the coefficient of subgrade reaction equal to 100 psi/in, and having the same radius of relative stiffness as the effective slab. The stresses in the effective slab are determined using the following equation:

$$\sigma_{eff}^{A}(P,\Delta T) = \frac{h_{eq}\gamma_{eff}}{h_{eff}\gamma_{eq}}\sigma_{eq}^{A}(P,\Delta T) \tag{25}$$

$$\sigma_{eff}^{A}(0,\Delta T) = \frac{h_{eq}\gamma_{eff}}{h_{eff}\gamma_{eq}}\sigma_{eq}^{A}(0,\Delta T) \tag{26}$$

$$\sigma_{eff}^{A}(P,0) = \frac{h_{eq}\gamma_{eff}}{h_{eff}\gamma_{eq}}\sigma_{eq}^{A}(P,0) \tag{27}$$

where γ_{eq} is the equivalent slab unit weight = 0.087 lb/in^2

<u>Step 10. Using NNB1, Compute Load-only Caused Stresses in the Equivalent Structure from the Wheels Located at the Mid-slab</u>

For single axle loading, stresses were computed from all the wheels in the axle. In the case of tandem or tridem axle loading, the wheels located away from the mid-slab were ignored, as shown in Figure 1.

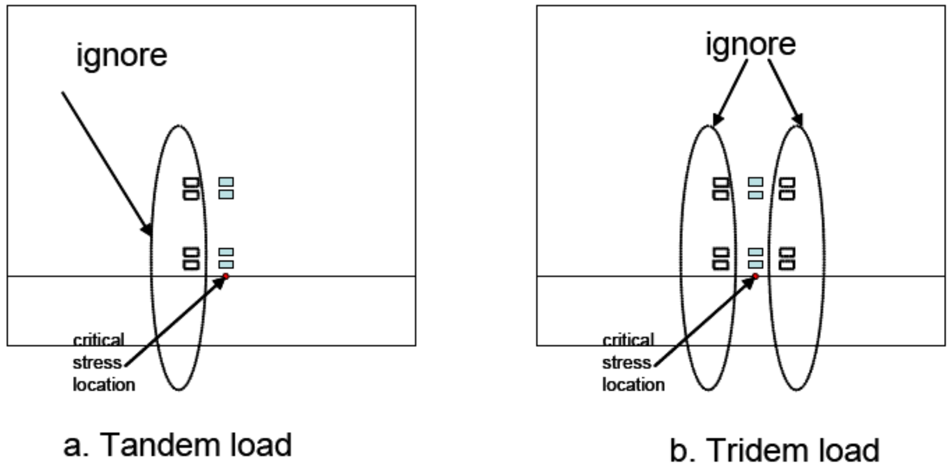

Figure 1. Analysis of tandem and tridem axle loading using NNB1.

Step 10.1 The stresses in the equivalent structure were computed with the assumption that there is no load transfer between the slabs in the system B (LTE=0). If the axle consists from dual tires, it was subdivided into two sub-axles as shown in Figure 2. The stresses were calculated separately from these sub-axles and the resulting stresses were superimposed to obtain $\sigma_{eq}^{B1}(0)$.

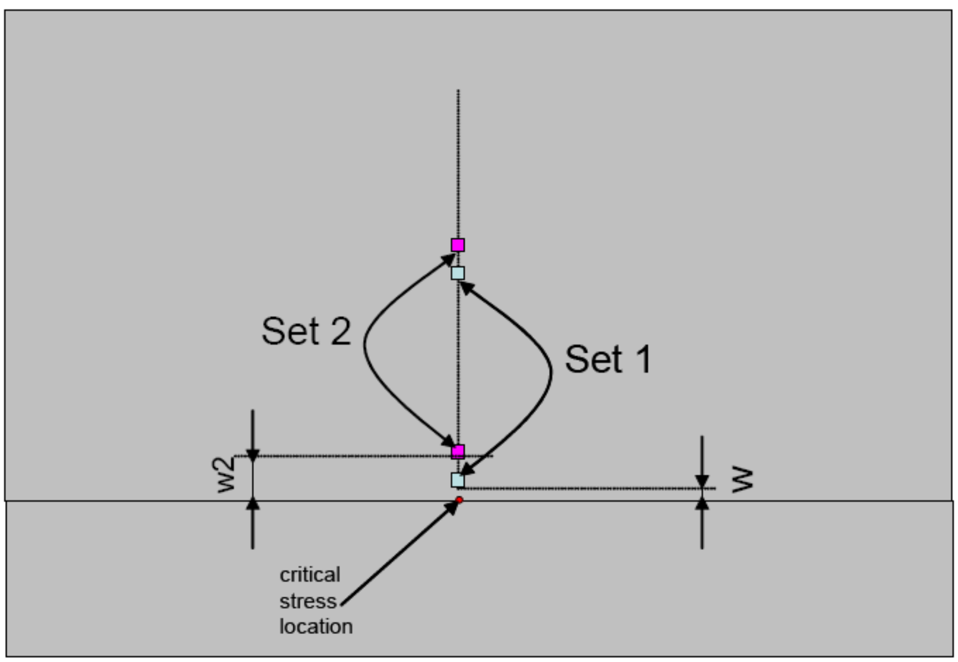

Figure 2. Analysis of a single axle load with dual tires using NNB1.

Step 10.2 The stresses in the equivalent structure were computed with the assumption that the load transfer efficiency between the two slabs in the system B is equal to shoulder LTE. If the axle consists from dual tires, it was subdivided into two sub-axles as shown in Figure 2. The stresses were calculated separately from these sub-axles and the resulting stresses were superimposed to obtain $\sigma_{eq}^{B1}(LTE_{sh})$.

Step 11 (only if tandem or tridem). Compute Stresses from the Remaining Wheels in the Axle using NNB2

Step 11.1 The stresses in the equivalent structure were computed with the assumption that there is no load transfer between the slabs in the system B (LTE=0). The stresses should be computed from the individual wheels (four for a tandem axle and eight for a tridem) and superimposed to obtain $\sigma_{eq}^{B2}(0)$.

Step 11.2 The stresses in the equivalent structure were computed with the assumption that the load transfer efficiency between two slabs in the system B is equal to shoulder LTE. The stresses should be computed from the individual wheels (four for a tandem axle and eight for a tridem) and superimposed to obtain $\sigma_{eq}^{B2}(LTE_{sh})$

Step 12. Determine Load-only Caused Stresses in the Equivalent Structure from the Entire Axle

- Single axle loading

$$\sigma_{eq}^{B}(0) = \sigma_{eq}^{B1}(0) \text{ and } \sigma_{eq}^{B}(LTE_{sh}) = \sigma_{eq}^{B1}(LTE_{sh}) \tag{28}$$

- Tandem or tridem laoding

$$\sigma_{eq}^{B}(0) = \sigma_{eq}^{B1}(0) + \sigma_{eq}^{B2}(0) \text{ and } \sigma_{eq}^{B}(LTE_{sh}) = \sigma_{eq}^{B1}(LTE_{sh}) + \sigma_{eq}^{B2}(LTE_{sh}) \tag{29}$$

Step 13. Determine Load-only Caused Stresses in the Effective Slab

The load-only causing stresses in the effective slab can be determined using the following expression:

$$\sigma_{eff}^{B}(0) = \frac{p_{eff}}{p_{eq}} \frac{h_{eq}^2}{h_{eff}^2} \sigma_{eq}^{B}(0) \tag{30}$$

$$\sigma_{eff}^{B}(LTE_{sh}) = \frac{p_{eff}}{p_{eq}} \frac{h_{eq}^2}{h_{eff}^2} \sigma_{eq}^{B}(LTE_{sh}) \tag{31}$$

where p_{eq} is the wheel pressure in the equivalent system = 100 psi, and p_{eff} is the actual wheel pressure,

Step 14. Find Stress Load Transfer Efficiency for the Given Axle Load Configuration and the Axle Load Position

$$LTE_{stress} = \frac{\sigma_{eff}^{B}(LTE_{sh})}{\sigma_{eff}^{B}(0)} \tag{32}$$

Step 15. Find Axle Loading Induced Component of Bending Stresses in the Effective Slab if the Shoulder Provides no Edge Support to the Traffic Lane Slab

The axle loading induced component of bending stress is the stress in the slab caused by the action of axle loading along with temperature curling and is given as:

$$\sigma_{load,noshoulder} = \sigma_{eff}^{A}(P,\Delta T) - \sigma_{eff}^{A}(0,\Delta T) - \sigma_{eff}^{A}(P,0) + \sigma_{eff}^{B}(0) \qquad (33)$$

Step 16. Find Axle Loading Induced Component of Bending Stresses Accounting for the Shoulder Edge Support to the Traffic Lane Slab

$$\sigma_{load,shoulder} = \sigma_{load,noshoulder} * LTE_{stress} \qquad (34)$$

Step 17. Find Combined Stress in the Effective Slab

$$\sigma_{comb} = \sigma_{load,shoulder} + \sigma_{curl} \qquad (35)$$

$$\sigma_{curl} = \sigma_{eff}^{A}(0,\Delta T) \qquad (36)$$

Step 18. Find Bending PCC Stresses

Bending stresses (i.e., stresses caused by an axle load and a linear component of the temperature distribution) at the bottom of the PCC slab can be found using the following relationship:

1. Fully bonded AC/PCC and fully bonded PCC/base interfaces

$$\sigma_{PCC,bend} = \frac{2*(h_{PCC} + h_{AC} - x)}{h_{eff}} \sigma_{comb} \qquad (37)$$

2. Unbonded AC/PCC and unbonded PCC/base interfaces

$$\sigma_{PCC,bend} = \frac{h_{PCC}}{h_{eff}} \sigma_{comb} \qquad (38)$$

3. Unbonded AC/PCC and fully bonded PCC/base interfaces

$$\sigma_{PCC,bend} = \frac{2*(h_{PCC} - x)}{h_{eff}} \sigma_{comb} \qquad (39)$$

4. Fully bonded AC/PCC and unbonded PCC/base interfaces

$$\sigma_{PCC,bend} = \frac{2*(h_{PCC} + h_{AC} - x)}{h_{eff}} \sigma_{comb} \qquad (40)$$

where: x is the depth of the N.A. from the top of the AC layer.

Step 19. Find Total PCC Stresses

$$\sigma_{PCC} = \sigma_{PCC,bend} + \sigma_{NL,PCC,bot} \qquad (41)$$

where σ_{PCC} is the total stress at the bottom of the PCC slab, $\sigma_{PCC,bend}$ is the bending stress at the bottom of the PCC slab, and $\sigma_{NL,PCC,bot}$ is the stress at the bottom of the PCC layer caused by the nonlinear strain component of the temperature distribution.

The neural networks and the computed critical PCC stresses were implemented into a FORTRAN program on guidelines similar to MEPDG. This program computes the stress for each hour of the pavement design life based on the hourly temperature distributions in the AC and PCC layer and traffic distributions obtained from MEPDG internal files.

REFERENCES

AASHTO 2010. “Mechanistic-Empirical Pavement Design Guide, Interim Edition: A Manual of Practice.” American Association of State Highway and Transportation Officials, Washington, DC.

Choubane, B., and M. Tia. (1992). “Nonlinear Temperature Gradient Effect on Maximum Warping Stresses in Rigid Pavements,” *Transportation Research Record 1370,* National Research Council, Washington, D.C., pp. 11-19.

Ioannides, A.M., L. Khazanovich, and J.L. Becque. (1992). “Structural Evaluation of Base Layers in Concrete Pavement Systems,” *Transportation Research Record 1370*, Washington, DC: Transportation Research Board, National Research Council, pp. 20-28.

Ioannides, A.M., and L. Khazanovich. (1998). “Nonlinear Temperature Effects in Multi-Layered Concrete Pavements,” *ASCE Journal of Transportation Engineering*, vol. 124, no. 2.

Khazanovich, L. (1994). “Structural Analysis of Multi-Layered Concrete Pavement Systems.” Ph.D. Dissertation, University of Illinois, Urbana, Illinois.

Mohamed, A.R., and W. Hansen. (1997). “Effect of Nonlinear Temperature Gradient on Curling Stresses in Concrete Pavements,” *Transportation Research Record 1568*, National Academy Press, Washington, D.C., pp. 65-71.

Thomlinson, J. (1940). “Temperature Variations and Consequent Stresses Produced by Daily and Seasonal Temperature Cycles in Concrete Slabs,” *Concrete Constructional Engineering*, Vol. 36, No. 6, pp. 298-307; No. 7, pp. 352-360.

APPENDIX A

Computation of the non-linear strain-causing temperature stresses using equivalency concepts for 3-layered system – Additional Cases

1. Unbonded AC/PCC and unbonded PCC/Base

For unbonded layers (full slip) with no horizontal constraint, the constant strain-causing temperature component causes free expansion of the layer. This free expansion in one layer does not affect the other layer as they are not bonded. It does not cause stress (and strain) in any of the other layers. Therefore, the layers can be treated independently of one another to compute the constant strain-causing temperature component. The neutral axis (N.A.) of the AC and the base layer in terms of the N.A. of the PCC layer is given as follows:

$$\zeta_{AC} = z + \left(\frac{h_{AC} + h_{PCC}}{2}\right)$$

$$\zeta_{Base} = z - \left(\frac{h_{PCC} + h_{Base}}{2}\right)$$

The constant strain-causing temperature components for each layer are:

$$T_{c,AC} = T_o + \frac{\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \alpha_{AC} E_{AC} \left[T(\zeta_{AC}) - T_o\right] d\zeta_{AC}}{\alpha_{AC} \int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} E_{AC}\, d\zeta_{AC}} = \frac{1}{h_{AC}} \int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} T(\zeta_{AC})\, d\zeta_{AC}$$

$$T_{c,PCC} = T_o + \frac{\int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} \alpha_{PCC} E_{PCC} \left[T(z) - T_o\right] dz}{\alpha_{PCC} \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} E_{PCC}\, dz} = \frac{1}{h_{PCC}} \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} T(z)\, dz$$

$$T_{c,Base} = T_o + \frac{\int_{\frac{-h_{Base}}{2}}^{\frac{h_{Base}}{2}} \alpha_{Base} E_{Base} \left[T(\zeta_{Base}) - T_o\right] d\zeta_{Base}}{\alpha_{Base} \int_{\frac{-h_{Base}}{2}}^{\frac{h_{Base}}{2}} E_{Base} d\zeta_{Base}} = \frac{1}{h_{Base}} \int_{\frac{-h_{Base}}{2}}^{\frac{h_{Base}}{2}} T(\zeta_{Base}) d\zeta_{Base}$$

The constant strain-causing temperature in the PCC layer can be approximated as:

$$T_{c,PCC} = \frac{1}{h_{PCC}} \frac{h_{PCC}}{20} \left(T_1 + 2 * \sum_{i=2}^{10} T_i + T_{11} \right)$$

Therefore,

$$T_{c,PCC} - T_o = \frac{1}{20} \left(T_1 + 2 * \sum_{i=2}^{10} T_i + T_{11} \right) - T_o$$

The linear strain-causing temperature component is:

$$T_L(z) = T_o + \frac{z}{\alpha(z)} \frac{\sum_{i=1}^{l} \int_h \alpha(z) E(z) \left[T(z) - T_o\right] z dz}{\sum_{i=1}^{l} \int_h E(z) z^2 dz}$$

$$T_{L,AC} = T_o + S_1 \frac{\zeta_{AC}}{\alpha_{AC}} = T_o + 12S \frac{\zeta_{AC}}{\alpha_{AC}}$$

$$T_{L,PCC} = T_o + S_1 \frac{z}{\alpha_{PCC}} = T_o + 12S \frac{z}{\alpha_{PCC}}$$

$$T_{L,Base} = T_o + S_1 \frac{\zeta_{Base}}{\alpha_{Base}} = T_o + 12S \frac{\zeta_{Base}}{\alpha_{Base}}$$

$$S_1 = \frac{\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \alpha_{AC} E_{AC} \left[T(\zeta_{AC}) - T_o\right] \zeta_{AC} d\zeta_{AC} + \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} \alpha_{PCC} E_{PCC} \left[T(z) - T_o\right] z dz + \int_{-\frac{h_{Base}}{2}}^{\frac{h_{Base}}{2}} \alpha_{Base} E_{Base} \left[T(\zeta_{Base}) - T_o\right] \zeta_{Base} d\zeta_{Base}}{\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} E_{AC} \zeta_{AC}^2 d\zeta_{AC} + \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} E_{PCC} z^2 dz + \int_{-\frac{h_{Base}}{2}}^{\frac{h_{Base}}{2}} E_{Base} \zeta_{Base}^2 d\zeta_{Base}}$$

$$S = \frac{\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \alpha_{AC} E_{AC} \left[T(\zeta_{AC}) - T_o\right] \zeta_{AC} d\zeta_{AC} + \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} \alpha_{PCC} E_{PCC} \left[T(z) - T_o\right] z dz + \int_{-\frac{h_{Base}}{2}}^{\frac{h_{Base}}{2}} \alpha_{Base} E_{Base} \left[T(\zeta_{Base}) - T_o\right] \zeta_{Base} d\zeta_{Base}}{E_{AC} h_{AC}^3 + E_{PCC} h_{PCC}^3 + E_{Base} h_{Base}^3}$$

For an equivalent slab with EPCC and αPCC, the effective thickness is given as:

$$h_{eff} = \sqrt[3]{\frac{E_{AC}}{E_{PCC}} h_{AC}^3 + h_{PCC}^3 + \frac{E_{Base}}{E_{PCC}} h_{Base}^3}$$

$$\gamma_{eff} = \frac{h_{AC}\gamma_{AC} + h_{PCC}\gamma_{PCC}}{h_{eff}}$$

Therefore, the linear strain-causing temperature can be written as:

$$T_{L,eff} = T_o + 12S \frac{z_{eff}}{\alpha_{PCC}}$$

$$S = \frac{\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \alpha_{AC} E_{AC} \left[T(\zeta_{AC}) - T_o\right] \zeta_{AC} d\zeta_{AC} + \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} \alpha_{PCC} E_{PCC} \left[T(z) - T_o\right] z dz + \int_{-\frac{h_{Base}}{2}}^{\frac{h_{Base}}{2}} \alpha_{Base} E_{Base} \left[T(\zeta_{Base}) - T_o\right] \zeta_{Base} d\zeta_{Base}}{E_{PCC} h_{eff}^3}$$

$$T_{L,eff,top} = T_o + 12S \frac{-h_{eff}/2}{\alpha_{PCC}}$$

$$T_{L,eff,bot} = T_o + 12S \frac{h_{eff}/2}{\alpha_{PCC}}$$

$$\Delta(T_{L,eff} - T_o) = -12S \frac{h_{eff}}{\alpha_{PCC}}$$

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2} \left(\frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \left[T(\zeta_{AC}) - T_o\right] \zeta_{AC} d\zeta_{AC} + \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} \left[T(z) - T_o\right] z dz + \frac{\alpha_{Base} E_{Base}}{\alpha_{PCC} E_{PCC}} \int_{-\frac{h_{Base}}{2}}^{\frac{h_{Base}}{2}} \left[T(\zeta_{Base}) - T_o\right] \zeta_{Base} d\zeta_{Base} \right)$$

As per the assumption, the temperature distribution in the base layer is constant and equal to the temperature at the bottom of the PCC layer. Also, the reference temperature, i.e., the temperature at which the slab is flat, is equal to the temperature at the bottom of the PCC layer.

$T(\zeta_{Base}) = T_{11}$ and $T_o = T_{11}$

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2}\left(\frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \left[T(\zeta_{AC}) - T_{11}\right]\zeta_{AC}\, d\zeta_{AC} + \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} \left[T(z) - T_{11}\right] z\, dz \right)$$

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2}\left(\frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} T(\zeta_{AC})\zeta_{AC}\, d\zeta_{AC} + \int_{-\frac{h_{PCC}}{2}}^{\frac{h_{PCC}}{2}} T(z) z\, dz \right)$$

This can be approximated as:

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2}\left(\begin{array}{l} \dfrac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \dfrac{h_{AC}}{24} \displaystyle\sum_{i=1}^{4} \left(T_i * \left((3i-2) * \frac{h_{AC}}{4} - 3x_{AC} \right) + T_{i+1} * \left((3i-1) * \frac{h_{AC}}{4} - 3x_{AC} \right) \right) \\ + \dfrac{h_{PCC}}{60} \displaystyle\sum_{i=1}^{10} \left(T_i * \left((3i-2) * \frac{h_{PCC}}{10} - 3x_{PCC} \right) + T_{i+1} * \left((3i-1) * \frac{h_{PCC}}{10} - 3x_{PCC} \right) \right) \end{array} \right)$$

where,

$x_{AC} = \dfrac{h_{AC}}{2}$ and $x_{PCC} = \dfrac{h_{PCC}}{2}$

Therefore, the linear strain-causing temperature at the bottom and the top of the PCC layer, respectively, can be written as:

$$T_{L,PCC,bot} - T_o = \frac{\Delta(T_{L,eff} - T_o)}{h_{eff}} \frac{h_{PCC}}{2}$$

$$T_{L,PCC,top} - T_o = -\frac{\Delta(T_{L,eff} - T_o)}{h_{eff}} \frac{h_{PCC}}{2}$$

2. Unbonded AC/PCC and bonded PCC/Base

x: From the top of PCC layer (AC – PCC interface)

$$x_{PB} = \frac{\dfrac{h_{PCC}^2}{2} + \dfrac{E_{Base}}{E_{PCC}} h_{Base}\left(h_{PCC} + \dfrac{h_{Base}}{2}\right)}{h_{PCC} + \dfrac{E_{Base}}{E_{PCC}} h_{Base}}$$

$$h_{eff} = \sqrt[3]{\frac{E_{AC}}{E_{PCC}} h_{AC}^3 + h_{PCC}^3 + \frac{E_{Base}}{E_{PCC}} h_{Base}^3 + 12\left[h_{PCC}\left(x_{PB} - \frac{h_{PCC}}{2}\right)^2 + \frac{E_{Base}}{E_{PCC}} h_{Base}\left(h_{PCC} + \frac{h_{Base}}{2} - x_{PB}\right)^2\right]}$$

$$\gamma_{eff} = \frac{h_{AC}\gamma_{AC} + h_{PCC}\gamma_{PCC} + h_{Base}\gamma_{Base}}{h_{eff}}$$

The constant strain-causing temperature component is:

$$T_{c,AC} = T_o + \frac{\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \alpha_{AC} E_{AC}\left[T(\zeta_{AC}) - T_o\right] d\zeta_{AC}}{\alpha_{AC}\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} E_{AC}\, d\zeta_{AC}} = \frac{1}{h_{AC}} \int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} T(\zeta_{AC})\, d\zeta_{AC}$$

$$T_c(z) = T_o + \frac{1}{\alpha(z)} \frac{\int_{-x_{PB}}^{h_{PCC}-x_{PB}} \alpha_{PCC} E_{PCC}\left[T(z) - T_o\right] dz + \int_{h_{PCC}-x_{PB}}^{h_{PCC}+h_{Base}-x_{PB}} \alpha_{Base} E_{Base}\left[T(z) - T_o\right] dz}{E_{PCC} h_{PCC} + E_{Base} h_{Base}}$$

For the PCC layer this can be approximated as:

$$T_{c,PCC} - T_o = \frac{E_{PCC}\left(\dfrac{h_{PCC}}{20}\left(T_1 + 2 * \sum_{i=2}^{10} T_i + T_{11}\right) - T_o h_{PCC}\right)}{E_{PCC} h_{PCC} + E_{Base} h_{Base}}$$

The linear strain-causing temperature component is:

$$T_L(z) = T_o + \frac{z}{\alpha(z)} \frac{\sum_{i=1}^{l} \int_h \alpha(z) E(z)\left[T(z) - T_o\right] z\, dz}{\sum_{i=1}^{l} \int_h E(z) z^2\, dz}$$

$$\zeta_{AC} = z + \left(\frac{h_{AC}}{2} + x_{PB} \right)$$

$$T_{L,AC} = T_o + 12S \frac{\zeta_{AC}}{\alpha_{AC}}$$

$$T_L(z) = T_o + 12S \frac{z}{\alpha(z)}$$

$$S = \frac{\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \alpha_{AC} E_{AC} \left[T(\zeta_{AC}) - T_o \right] \zeta_{AC} d\zeta_{AC} + \int_{-x_{PB}}^{h_{PCC} - x_{PB}} \alpha_{PCC} E_{PCC} \left[T(z) - T_o \right] zdz + \int_{h_{PCC} - x_{PB}}^{h_{PCC} + h_{Base} - x_{PB}} \alpha_{Base} E_{Base} \left[T(z) - T_o \right] zdz}{E_{AC} h_{AC}^3 + E_{PCC} h_{PCC}^3 + E_{Base} h_{Base}^3 + 12 \left[E_{PCC} h_{PCC} \left(x_{PB} - \frac{h_{PCC}}{2} \right)^2 + E_{Base} h_{Base} \left(h_{PCC} + \frac{h_{Base}}{2} - x_{PB} \right)^2 \right]}$$

$$\Delta(T_{L,eff} - T_o) = \frac{-12 h_{eff}}{\alpha_{PCC}} \left(\frac{\int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \alpha_{AC} E_{AC} \left[T(\zeta_{AC}) - T_o \right] \zeta_{AC} d\zeta_{AC} + \int_{-x_{PB}}^{h_{PCC} - x_{PB}} \alpha_{PCC} E_{PCC} \left[T(z) - T_o \right] zdz + \int_{h_{PCC} - x_{PB}}^{h_{PCC} + h_{Base} - x_{PB}} \alpha_{Base} E_{Base} \left[T(z) - T_o \right] zdz}{E_{PCC} h_{eff}^3} \right)$$

$$T_{Base}(z) = T_{11} = T_o$$

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2} \left(\frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} \left[T(\zeta_{AC}) - T_{11} \right] \zeta_{AC} d\zeta_{AC} + \int_{-x_{PB}}^{h_{PCC} - x_{PB}} \left[T(z) - T_{11} \right] zdz \right)$$

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2} \left(\frac{\alpha_{AC} E_{AC}}{\alpha_{PCC} E_{PCC}} \int_{-\frac{h_{AC}}{2}}^{\frac{h_{AC}}{2}} T(\zeta_{AC}) \zeta_{AC} d\zeta_{AC} + \int_{-x_{PB}}^{h_{PCC} - x_{PB}} T(z) zdz - T_{11} \int_{-x_{PB}}^{h_{PCC} - x_{PB}} zdz \right)$$

This can be approximated as:

$$\Delta(T_{L,eff}-T_o)=\frac{-12}{h_{eff}^2}\left(\begin{array}{l}\dfrac{\alpha_{AC}E_{AC}}{\alpha_{PCC}E_{PCC}}\dfrac{h_{AC}}{24}\sum_{i=1}^{4}\left(T_i*\left((3i-2)*\dfrac{h_{AC}}{4}-3x_{AC}\right)+T_{i+1}*\left((3i-1)*\dfrac{h_{AC}}{4}-3x_{AC}\right)\right)\\+\dfrac{h_{PCC}}{60}\sum_{i=1}^{10}\left(T_i*\left((3i-2)*\dfrac{h_{PCC}}{10}-3x_{PB}\right)+T_{i+1}*\left((3i-1)*\dfrac{h_{PCC}}{10}-3x_{PB}\right)\right)\\-\dfrac{T_{11}}{2}h_{PCC}\left(h_{PCC}-2x_{PB}\right)\end{array}\right)$$

Therefore, the linear strain-causing temperature at the bottom and the top of the PCC layer, respectively, can be written as:

$$T_{L,PCC,bot}-T_o=\frac{\Delta(T_{L,eff}-T_o)}{h_{eff}}(h_{PCC}-x_{PB})$$

$$T_{L,PCC,top}-T_o=-\frac{\Delta(T_{L,eff}-T_o)}{h_{eff}}x_{PB}$$

3. Bonded AC/PCC and unbonded PCC/Base

x_{AP}: From the top of AC layer

$$x_{AP}=\frac{\dfrac{E_{AC}}{E_{PCC}}\dfrac{h_{AC}^2}{2}+h_{PCC}\left(h_{AC}+\dfrac{h_{PCC}}{2}\right)}{\dfrac{E_{AC}}{E_{PCC}}h_{AC}+h_{PCC}}$$

$$h_{eff}=\sqrt[3]{\frac{E_{AC}}{E_{PCC}}h_{AC}^3+h_{PCC}^3+12\left[\frac{E_{AC}}{E_{PCC}}h_{AC}\left(x_{AP}-\frac{h_{AC}}{2}\right)^2+h_{PCC}\left(h_{AC}+\frac{h_{PCC}}{2}-x_{AP}\right)^2\right]+\frac{E_{Base}}{E_{PCC}}h_{Base}^3}$$

$$\gamma_{eff}=\frac{h_{AC}\gamma_{AC}+h_{PCC}\gamma_{PCC}}{h_{eff}}$$

The constant strain-causing temperature component is:

$$T_c(z)=T_o+\frac{1}{\alpha(z)}\frac{\int_{-x_{AP}}^{h_{AC}-x_{AP}}\alpha_{AC}E_{AC}\left[T(z)-T_o\right]dz+\int_{h_{AC}-x_{AP}}^{h_{AC}+h_{PCC}-x_{AP}}\alpha_{PCC}E_{PCC}\left[T(z)-T_o\right]dz}{E_{AC}h_{AC}+E_{PCC}h_{PCC}}$$

This can be approximated as:

$$T_c(z)-T_o=\frac{\frac{\alpha_{AC}E_{AC}}{\alpha_{PCC}}\left(\frac{h_{AC}}{8}\left(T_{AC1}+2*\sum_{i=2}^{4}T_{ACi}+T_{AC5}\right)-T_o h_{AC}\right)+E_{PCC}\left(\frac{h_{PCC}}{20}\left(T_1+2*\sum_{i=2}^{10}T_i+T_{11}\right)-T_o h_{PCC}\right)}{E_{AC}h_{AC}+E_{PCC}h_{PCC}}$$

The linear strain-causing temperature component is:

$$T_L(z)=T_o+\frac{z}{\alpha(z)}\frac{\sum_{i=1}^{l}\int_h \alpha(z)E(z)[T(z)-T_o]zdz}{\sum_{i=1}^{l}\int_h E(z)z^2dz}$$

$$\zeta_{Base}=z-\left(h_{AC}+h_{PCC}-x_{AP}+\frac{h_{Base}}{2}\right)$$

$$T_L(z)=T_o+12S\frac{z}{\alpha(z)}$$

$$T_{L,Base}=T_o+12S\frac{\zeta_{Base}}{\alpha_{Base}}$$

$$S=\frac{\int_{-x_{AP}}^{h_{AC}-x_{AP}}\alpha_{AC}E_{AC}[T(z)-T_o]zdz+\int_{h_{AC}-x_{AP}}^{h_{AC}+h_{PCC}-x_{AP}}\alpha_{PCC}E_{PCC}[T(z)-T_o]zdz+\int_{-\frac{h_{Base}}{2}}^{\frac{h_{Base}}{2}}\alpha_{Base}E_{Base}[T(\zeta_{Base})-T_o]\zeta_{Base}d\zeta_{Base}}{E_{AC}h_{AC}^3+E_{PCC}h_{PCC}^3+12\left[E_{AC}h_{AC}\left(x_{AP}-\frac{h_{AC}}{2}\right)^2+E_{PCC}h_{PCC}\left(h_{AC}+\frac{h_{PCC}}{2}-x_{AP}\right)^2\right]+E_{Base}h_{Base}^3}$$

$$\Delta(T_{L,eff}-T_o)=\frac{-12h_{eff}}{\alpha_{PCC}}\left(\frac{\int_{-x_{AP}}^{h_{AC}-x_{AP}}\alpha_{AC}E_{AC}[T(z)-T_o]zdz+\int_{h_{AC}-x_{AP}}^{h_{AC}+h_{PCC}-x_{AP}}\alpha_{PCC}E_{PCC}[T(z)-T_o]zdz+\int_{-\frac{h_{Base}}{2}}^{\frac{h_{Base}}{2}}\alpha_{Base}E_{Base}[T(\zeta_{Base})-T_o]\zeta_{Base}d\zeta_{Base}}{E_{PCC}h_{eff}^3}\right)$$

$$T_{Base}(z)=T_{11}=T_o$$

$$\Delta(T_{L,eff}-T_o)=\frac{-12}{h_{eff}^2}\left(\int_{-x_{AP}}^{h_{AC}-x_{AP}}\frac{\alpha_{AC}E_{AC}}{\alpha_{PCC}E_{PCC}}[T(z)-T_{11}]zdz+\int_{h_{AC}-x_{AP}}^{h_{AC}+h_{PCC}-x_{AP}}[T(z)-T_{11}]zdz\right)$$

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2}\left(\frac{\alpha_{AC}E_{AC}}{\alpha_{PCC}E_{PCC}} \int_{-x_{AP}}^{h_{AC}-x_{AP}} T(z)zdz - \frac{\alpha_{AC}E_{AC}}{\alpha_{PCC}E_{PCC}} T_{11} \int_{-x_{AP}}^{h_{AC}-x_{AP}} zdz + \int_{h_{AC}-x_{AP}}^{h_{AC}+h_{PCC}-x_{AP}} T(z)zdz - T_{11} \int_{h_{AC}-x_{AP}}^{h_{AC}+h_{PCC}-x_{AP}} zdz \right)$$

This can be approximated as:

$$\Delta(T_{L,eff} - T_o) = \frac{-12}{h_{eff}^2}\left(\begin{array}{l} \frac{\alpha_{AC}E_{AC}}{\alpha_{PCC}E_{PCC}} \frac{h_{AC}}{24} \sum_{i=1}^{4} \left(T_i * \left((3i-2) * \frac{h_{AC}}{4} - 3x_{AP} \right) + T_{i+1} * \left((3i-1) * \frac{h_{AC}}{4} - 3x_{AP} \right) \right) \\ - \frac{\alpha_{AC}E_{AC}}{\alpha_{PCC}E_{PCC}} \frac{T_{11}}{2} h_{AC}\left(h_{AC} - 2x_{AP}\right) \\ + \frac{h_{PCC}}{60} \sum_{i=1}^{10} \left(T_i * \left((3i-2) * \frac{h_{PCC}}{10} - 3(x_{AP} - h_{AC}) \right) + T_{i+1} * \left((3i-1) * \frac{h_{PCC}}{10} - 3(x_{AP} - h_{AC}) \right) \right) \\ - \frac{T_{11}}{2} h_{PCC}\left(h_{PCC} + 2h_{AC} - 2x_{AP}\right) \end{array} \right)$$

Therefore, the linear strain-causing temperature at the bottom and the top of the PCC layer, respectively, can be written as:

$$T_{L,PCC,bot} - T_o = \frac{\Delta(T_{L,eff} - T_o)}{h_{eff}}(h_{PCC} + h_{AC} - x_{AP})$$

$$T_{L,PCC,top} - T_o = -\frac{\Delta(T_{L,eff} - T_o)}{h_{eff}}(x - h_{AC})$$